S.E (MECHANICAL ENGG) (SEM III) (CBGS)

Applied Mathematics III

31st May, 2014

3.00 pm to 6.00 pm

QP Code : NP-18610

(3 Hours)

[Total Marks :80

N.B. (1) Question no. **1** is **compulsory.**

(2) Solve any **three** questions out of the remaining Q.no. **2** to Q. no. **6.**

1. (a) Find the inverse Laplace transform of

$$\frac{S^2+5}{(S^2+4S+13)^2}$$

(b) If $V = 3x^2y + 6xy - y^3$, show that the function V is harmonic, find the corresponding analytic function.

(c) Evaluate $\int_C \bar{z}dz$ where C is the upper half of the circle $r = 1$.

(d) Prove that $f_1(x) = 1,\ f_2(x) = x,\ f_3(x) = \frac{3x^2-1}{2}$ are orthogonal over $(-1, 1)$.

2. (a) Evaluate $\int_0^\infty \frac{\cos at - \cos bt}{t}\,dt$

(b) Obtain complex form of fourier series $f(x) = e^{-ax}$ for in $(-\pi, \pi)$

(c) Using Crank-Nicholson simplified formula solve, $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$

$u(0,t) = 0,\ u(4,t) = 0,\ u(x,0) = \frac{x}{3}(16 - x^2)$

Find uij for i = 0, 1, 2, 3, 4, and j = 0, 1, 2.

3. (a) Evaluate $\int_C \frac{\sin^6 z}{(z - \pi/6)^3}\,dz$ where C is $|z| = 1$

(b) Find the fourier expansion for $f(x) = x - x^2 \quad -1 < x < 1$

(c) Determine the solution of one dimensional heat equation, $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary conditions $u(0,t) = 0 \quad u(\ell, t) = 0$ and $u(x,0) = x, \quad (0 < x < \ell)$, ℓ being length of the rod.

4. (a) Find inverse Laplace transform by using convolution theroem,

$$f(s) = \frac{s^2}{(s^2 - a^2)^2}$$

(b) Find the image of the region bounded by $x = 0$, $x = 2$, $y = 0$, $y = 2$ in the Z plane under transformation $W = (1 + i) Z$.

(c) Find all possible Laurent's expansions of the function $f(z) = \frac{7z - 2}{z(z-2)(z+1)}$ about $Z = -1$.

5. (a) Solve $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method, subject to the conditions $u(0, t) = 0$, $u(x, 0) = 0$, $u(1, t) = t$ taking $h = 0.25$, $0 < x < 1$.

(b) Obtain hlaf range sine series for $f(x)$ when

$$f(x) = x, \quad 0 < x < \frac{\pi}{2}$$

$$= \pi - x, \quad \frac{\pi}{2} < x < \pi$$

(c) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$ by using residues. $a > 0$, $b > 0$

6. (a) Find the orthogonal tranjectory of the family of curves $x^3y - xy^3 = c$.

(b) Obtain the fourier expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval

$0 < x < 2\pi$, $f(x + 2\pi) = f(x)$

Also deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots$

(c) Solve using Laplace transform $(D^2 - 3D + 2) y = 4 e^{2t}$, with $y(0) = -3$ $y'(0) = 5$.