

- N.B.** 1) Question No. 1 is compulsory.  
2) Answer **any Three** from remaining  
3) Figures to the right indicate full marks



1. a) Verify Laplace equation for  $u = \left(r + \frac{a^2}{r}\right) \cos \theta$ . 5  
b) Find Laplace transform of  $f(t) = e^{-3t} \sin 2t \cdot \cos 3t$ . 5  
c) Obtain Fourier series for  $f(x) = x$  in  $(-\pi, \pi)$ . 5  
d) Evaluate  $\int_C (z^2 + 3z^{-4}) dz$  where C is the upper half of the unit circle from (1,0) to (-1,0). 5
2. a) Obtain the Taylor's and Laurent series which represent the function  
 $f(z) = \frac{z}{(z+1)(z-2)}$  in the regions, i)  $|z| < 1$  ii)  $1 < |z| < 2$  6  
b) Obtain Complex form of Fourier series for  $f(x) = \cos hx$  in  $(-\pi, \pi)$  6  
c) Using Laplace transform, solve the differential equation:  
 $\frac{dx}{dt} + 2x = \sin \omega t$ , with  $x(0) = 1$ . 8
3. a) Solve  $\frac{\partial^2 u}{\partial x^2} - 100 \frac{\partial u}{\partial t} = 0$  with  $u(0, t) = 0, u(1, t) = 0, u(x, 0) = x(1 - x)$   
taking  $h = 0.1$  for three time steps up to  $t = 1.5$  by Bender - Schmidt method. 6  
b) Find the bilinear transformation which maps the points  $z = 1, i, -1$  into the points  $w = 0, 1, \infty$ . 6  
c) Obtain half range Fourier sine series for  $f(x) = \begin{cases} x, & 0 < x \leq \pi/2 \\ \pi - x, & \pi/2 \leq x < \pi \end{cases}$  8  
Hence, prove that -

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

[TURN OVER]

4. a) Find the orthogonal trajectory of the family of curves  $2x - x^3 + 3xy^2 = c$  6  
 b) Find the Fourier series for  $f(x) = x|x|$  in  $(-1, 1)$ . 6  
 c) Find the inverse Laplace transform of:-  
 i)  $F(s) = \frac{1}{s(s^2+16)}$ , using Convolution theorem, ii)  $F(s) = \cot^{-1}(s+1)$ . 8
5. a) Solve by Crank –Nicholson simplified formula  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ ,  
 $u(0, t) = 0$ ,  $u(5, t) = 100$ ,  $u(x, 0) = 20$  taking  $h = 1$  for one-time step. 6  
 b) Find the image of the circle  $|z| = 4$  in the  $z$ -plane under the transformation  
 $w = z + 4 + 3i$ . Draw the sketch. 6  
 c) Find the analytic function  $f(z) = u + iv$  if  

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2 \cos x - e^y - e^{-y}}$$
 when  $f\left(\frac{\pi}{2}\right) = 0$ . 8
6. a) Using Residue theorem, evaluate,  $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos \theta}$  6  
 b) Using Laplace transform, evaluate  $\int_0^{\infty} e^{-t}(1 + 3t + t^2)H(t - 2)dt$  6  
 c) A tightly stretched string with fixed end points  $x = 0$  and  $x = l$ , in the shape defined by  $y = kx(l - x)$  where  $k$  is a constant is released from this position of rest. Find  $y(x, t)$ , the vertical displacement if  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ . 8