Q. P. Code: 11701

(3hours) [Total marks: 80]

N.B. (1) Question No. 1 is compulsory.
(2) Answer **any Three** from remaining
(3) Figures to the right indicate full marks.

1. (a) Find Laplace transform of $e^{-4t} \sin ht \sin t$. 5

(b) Does there exist an analytic function whose real part is $x^3 - 3x^2y - y^3$. Give justification. 5

(c) Show that $\{\cos x, \cos 2x, \cos 3x, \ldots\ldots\}$ is a set of orthogonal functions over an interval $(-\pi, \pi)$. 5

(d) Evaluate $\int_0^{2+i} z^2 dz$ along the line joining the point $z_1 = 0$ and $z_2 = 2+i$. 5

2. (a) Obtain the Taylor's and Laurent series which represent the function,
$f(z) = \dfrac{1}{(z+1)(z+3)}$ valid in the regions,
(i) $|z| < 1$ (ii) $1 < |z| < 3$ (iii) $|z| > 3$ 6

(b) Find the bilinear transformation which maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$. 6

(c) Using Laplace transform, solve the differential equation :
$\dfrac{d^2x}{dt^2} + 4x = t$ with $x(0) = 1,\quad x'(0) = -2$ 8

3. (a) Solve $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial u}{\partial t} = 0$ by Bender –Schmidt method, given
$u(0,t) = 0, u(x,0) = x(4-x), u(4,t) = 0$, assuming $h = 1$, find u upto t=5. 6

(b) Using convolution theorem find the inverse Laplace transform of
$\dfrac{s}{(s^2+1)(s^2+4)}$. 6

(c) Determine the solution of one-dimensional heat equation $\dfrac{\partial u}{\partial t} = c^2\dfrac{\partial^2 u}{\partial x^2}$ under boundary condition $u(0,t) = u(l,t) = 0,\ u(x,0) = x,\ l$ being the length of rod. 8

[TURN OVER]

4. (a) Using Residue theorem, evaluate, $\int_0^{2\pi} \frac{d\theta}{5 + 3\sin\theta}$. 6

(b) Find the inverse Laplace transform of the following:

$$\frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$$ 6

(c) Obtain Half Range Sine Series of $f(x) = x(\pi - x)$ in $(0, \pi)$.

Hence, evaluate $-\sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)^3}$.

8

5. (a) If $f(x) = e^{-3x}$, $-1 < x < 1$. Obtain Complex form of $f(x)$ in $(-1, 1)$. 6

(b) Find the orthogonal trajectory of the family of curves $3x^2y - y^3 = c$. 6

(c) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,

$u(0,t) = 0,\ u(1,t) = 2t,\ \cancel{u = 0}$, for two time steps taking $h = 0.25$. 8

$u(x,0) = 0$

6. (a) Obtain the Fourier series for $f(x)$ where

$$f(x) = x + \frac{\pi}{2} \qquad -\pi < x < 0$$

$$= \frac{\pi}{2} - x \qquad 0 < x < \pi$$ 6

(b) Prove that $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$ 6

(c) Find bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -1$. Hence, find the image of $|z| \leq 1$ onto the w-plane. 8
