N.B. : (1) Question No. **1** is **Compulsory**.

(2) **Answer** any **Three** from remaining.

(3) **Figures** to the **right** indicate **marks**.

1. (a) Find laplace of $\sin \sqrt{t}$ 5

    (b) Show that the set of functions $\sin\left(\frac{\pi x}{2L}\right)$, $\sin\left(\frac{3\pi x}{2L}\right)$, $\sin\left(\frac{5\pi x}{2L}\right)$ is orthogonal over   5

         (O, L).

    (c) Show that u = sinx cos hy + 2 cos x sin hy + x²-y² + 4xy Satisfies laplace equation   5
         and find its corresponding analytic function f(z) = u + iv.

    (d) Determine constants a,b,c,d if $f(z) = x^2+2axy+by^2 + i(cx^2+2dxy+y^2)$ is analytic.   5

2. (a) Find complex form of fourier series $f(x) = e^{3x}$ in 0<x<3.   6

    (b) Using Crank Nicholson Method solve $u_t = u_{xx}$ subject to u(x,0) = 0 u(0, t)=0 and   6
         u(1,t) = t for two time steps.

    (c) Solve using laplace transforms $\frac{d^2y}{dt^2} + y = t$, $y(0) = 1$, $y^1(0) = 0$.   8

3. (a) Find bilinear transformation that maps the points 0,1-∞ of the z plane into -5, -1,   6
         3 of w plane.

    (b) By using Convolution Theorem find inverse laplace transform of $\frac{1}{(S^2 +4S+13)^2}$ 6

    (c) Find fourier series of $f(x) = x^2$ $(-\pi \le x \le \pi)$ and prove that   8

         (i) $\frac{\pi^2}{6} = \sum_1^\infty \frac{1}{n^2}$

         (ii) $\frac{\pi^2}{12} = \sum_1^\infty \frac{(-1)^{n+1}}{n^2}$

         (iii) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$

4. (a) Evaluate $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt$ 6

    (b) Solve $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$ by   6

Bender schmidt method subject to conditions u(o, t) = 0 u (x, 0) = 0
u(l, t) = t taking h = 0.25 0 < x < 1

(c) Obtain two distinct Laurent's Series for $f(z) = \dfrac{2z-3}{Z^2 - 4z - 3}$ in Powers of (z-4)

　　indicating Region of Convergence.　　　　　　　　　　　　　　　　　　8

5. (a) Evaluate $\int\limits_{0}^{1+i} Z^2 dZ$ along　　　　　　　　　　　　　　　　6

　　(i) line y = x
　　(ii) Parabola $x = y^2$
　　Is line independent of path? Explian.

(b) Find half range Cosine Series for $f(x) = e^x$ 0 < x < 1.　　　　6

(c) Find analytic function　　　　　　　　　　　　　　　　　　　　8
　　f(z) = u + iv such that

$$u - v = \frac{Cosx + Sinx - e^{-y}}{2Cosx - e^{y} - e^{-y}}$$

　　when $f\left(\frac{\pi}{2}\right) = 0$

6. (a) A tightly streched sting with fixed end points x = 0 and x = ℓ in the shape defined　　6
　　by y = K x (l - x) where K is a Constant is released from this position of rest. Find y(x,t)
　　The vertical displacement

　　if $\dfrac{\partial^2 y}{\partial t^2} = C^2 \dfrac{\partial^2 y}{\partial x^2}$

(b) Find image of region bounded by x = 0, x = 2 y = 0 y = 2 in the z plane under the　　6
　　transformation w = (1 + i) Z

(c) Evaluate $\int\limits_{0}^{2\pi} \dfrac{d\theta}{25 - 16\cos^2\theta}$　　　　　　　　　　　　8