

- N.B. 1) Question No. 1 is compulsory.  
2) Answer **any Three** from remaining  
3) Figures to the right indicate full marks

1. a) State Cauchy Reimann equation in polar form. Use them to find  $p$  if  $f(z) = r^2 \cos 2\theta + i \sin p\theta$  is analytic. 5

b) Find Laplace transform of  $f(t) = te^{-3t} \sin t$ . 5

c) Find half-range sine series for  $f(x) = \frac{\pi}{4}$  in  $(0, \pi)$ . Hence, show that 5

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

d) Evaluate  $\int_C (z - z^3) dz$ , where  $C$  is left half of the unit circle from  $-i$  to  $i$ . 5

2. a) Obtain the Taylor's and the Laurent series which represent the function  $f(z) = \frac{2}{(z-1)(z-2)}$  in the regions, i)  $|z| < 1$  ii)  $1 < |z| < 2$  6

b) Obtain complex form of Fourier series of  $f(x) = e^{-x}$ ,  $-1 < x < 1$  in  $(-1, 1)$ . 6

c) Using Laplace transform, solve the differential equation,  $\frac{dx}{dt} + 2x = \cos \omega t$ , with  $x(0) = 0$ . 8

3. a) Solve  $\frac{\partial^2 u}{\partial x^2} - 100 \frac{\partial u}{\partial t} = 0$  with  $u(0, t) = 0, u(1, t) = 0, u(x, 0) = x(1-x)$  taking  $h = 0.1$  for three time steps up to  $t = 1.5$  by Bender-Schmidt method. 6

b) Find the bilinear transformation which maps the points  $z = 0, -1, i$  into the points  $w = i, 0, \infty$ . 6

c) Obtain Fourier Series of  $f(x) = \begin{cases} x, & 0 < x \leq \pi \\ 2\pi - x, & \pi \leq x < 2\pi \end{cases}$  in  $(0, 2\pi)$  8

Hence, deduce that -

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

[TURN OVER]

4. a) Find the orthogonal trajectory of the family of curves  $2x - x^3 + 3xy^2 = c$  6  
 b) Find the Fourier series for  $f(x) = 1 - x^2$  in  $(-1, 1)$ . 6  
 c) Find the inverse Laplace transform of:  
 i)  $F(s) = \frac{1}{s(s^2+9)}$ , using Convolution theorem, ii)  $F(s) = \cot^{-1}(s+1)$ . 8
5. a) Solve by Crank-Nicholson simplified formula  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ ,  
 $u(0, t) = 0, u(5, t) = 100, u(x, 0) = 20$  taking  $h = 1$  for one-time step. 6  
 b) Find the image of the circle  $|z| = 4$  in the  $z$ -plane under the transformation  
 $w = z + 2 + 3i$ . Draw the sketch. 6  
 c) If  $v = 3x^2y + 6xy - y^3$ , show that  $v$  is harmonic and find the corresponding  
 analytic function  $f(z) = u + iv$ . 8
6. a) Using Residue theorem, evaluate,  $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos \theta}$  6  
 b) Using Laplace transform, evaluate  $\int_0^{\infty} e^{-t}(1 + 3t + t^2)H(t - 2)dt$  6  
 c) A tightly stretched string with fixed end points  $x = 0$  and  $x = l$ , in the shape  
 defined by  $y = kx(l - x)$  where  $k$  is a constant, is released from this position  
 of rest. Find  $y(x, t)$ , the vertical displacement if  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ . 8