

(3hours)

[Total marks: 80]

- N.B.** (1) Question No. 1 is compulsory.  
 (2) Answer **any Three** from remaining  
 (3) Figures to the right indicate full marks.

1. (a) Find Laplace transform of  $e^{-4t} \sin ht \sin t$ . 5  
 (b) Does there exist an analytic function whose real part is  $x^3 - 3x^2y - y^3$ . Give justification. 5  
 (c) Show that  $\{\cos x, \cos 2x, \cos 3x, \dots\}$  is a set of orthogonal functions over an interval  $(-\pi, \pi)$ . 5  
 (d) Evaluate  $\int_0^{2+i} z^2 dz$  along the line joining the point  $z_1 = 0$  and  $z_2 = 2 + i$ . 5
2. (a) Obtain the Taylor's and Laurent series which represent the function,  
 $f(z) = \frac{1}{(z+1)(z+3)}$  valid in the regions,  
 (i)  $|z| < 1$  (ii)  $1 < |z| < 3$  (iii)  $|z| > 3$  6  
 (b) Find the bilinear transformation which maps the points  $z = \infty, i, 0$  into the points  $w = 0, i, \infty$ . 6  
 (c) Using Laplace transform, solve the differential equation :  
 $\frac{d^2x}{dt^2} + 4x = t$  with  $x(0) = 1, x'(0) = -2$  8
3. (a) Solve  $\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial u}{\partial t} = 0$  by Bender-Schmidt method, given  
 $u(0, t) = 0, u(x, 0) = x(4-x), u(4, t) = 0$ , assuming  $h = 1$ , find  $u$  upto  $t=5$ . 6  
 (b) Using convolution theorem find the inverse Laplace transform of  
 $\frac{s}{(s^2+1)(s^2+4)}$  6  
 (c) Determine the solution of one-dimensional heat equation  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$   
 under boundary condition  $u(0, t) = u(l, t) = 0, u(x, 0) = x$ ,  $l$  being the length of rod. 8

[TURN OVER]

4. (a) Using Residue theorem, evaluate,  $\int_0^{2\pi} \frac{d\theta}{5 + 3\sin \theta}$ . 6

(b) Find the inverse Laplace transform of the following:

$$\frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$$
 6

(c) Obtain Half Range Sine Series of  $f(x) = x(\pi - x)$  in  $(0, \pi)$ .

Hence, evaluate  $-\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^4}$ .

8

5. (a) If  $f(x) = e^{-1x}$ ,  $-1 < x < 1$ . Obtain Complex form of  $f(x)$  in  $(-1, 1)$ . 6

(b) Find the orthogonal trajectory of the family of curves  $3x^2y - y^3 = c$ . 6

(c) Solve by Crank-Nicholson simplified formula  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ ,

$$u(0, t) = 0, u(1, t) = 2t, u = 0, \text{ for two time steps taking } h = 0.25. \quad 8$$

6. (a) Obtain the Fourier series for  $f(x)$  where

$$f(x) = x + \frac{\pi}{2} \quad -\pi < x < 0$$

$$= \frac{\pi}{2} - x \quad 0 < x < \pi$$
 6

(b) Prove that  $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$  6

(c) Find bilinear transformation which maps the points  $z = 1, i, -1$  onto the points  $w = i, 0, -1$ . Hence, find the image of  $|z| \leq 1$  onto the  $w$ -plane. 8