SE Mech / MECA | Sem-III Sub:- AM-III ~~Auto~~ -
mechatronics (CBSGS) Date:- 25-11-16

Q.P. Code : 538301

(3 Hours) **[Total Marks : 80**

N.B. (1) Question No.1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Figures to right indicate full marks.

1 a) Evaluate $\int_C (\bar{z} + 2z)dz$ along the circle $x^2 + y^2 = 1$ (5)

b) Evaluate the integral using Laplace Transform $\int_0^\infty e^{-t}\left(t\sqrt{1+\sin t}\right) dt$ (5)

c) Determine the analytic function whose real part is $u = -r^3 \sin 3\theta$. (5)

d) A rod of length l has its ends A and B kept at $0^\circ C$ and $100^\circ C$ respectively until steady state conditions prevail. If the temperature at B is reduced suddenly to $0^\circ C$ and kept so while that of A is maintained. Find the temperature $u(x,t)$ at a distance from A and at time t. (5)

2 a) Find complex form of Fourier series of $f(x) = e^{2x}$ in (0,2) (6)

b) Find the orthogonal trajectory of the family of curves given by $2x - x^3 + 3xy^2 = a$ (6)

c) Using Bender Schmidt method solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ subject to the conditions $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = \sin \pi x$, $0 \le x \le 1$. Assume h=0.2 (8)

3 a) Find k such that $\frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}(\frac{kx}{y})$ is analytic (6)

b) Evaluate $\int_C \frac{1}{\left(z^3 - 1\right)^2} dz$ where C is the circle $|z - 1| = 1$ (6)

c) Show that the set of functions $\left\{Sin\left(\frac{\pi x}{2L}\right), Sin\left(\frac{3\pi x}{2L}\right), Sin\left(\frac{5\pi x}{2L}\right), \ldots\ldots\right\}$ forms an orthogonal set over the interval $[0, L]$. Construct corresponding orthonormal set. (8)

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4 a) Find Laplace Transform of the periodic function (6)

$$f(t)=\begin{cases}\sin 2t\ , & 0<t<\dfrac{\pi}{2}\\ 0, & \dfrac{\pi}{2}<t<\pi\end{cases}\quad f(t)=(t+\pi)$$

b) Find half range sine series for $x\sin x$ in $(0,\pi)$ (6)

c) Expand $f(z)=\dfrac{z^2-1}{z^2+5z+6}$ around $z=1$ (8)

5 a) Using residue theorem evaluate $\oint_C \dfrac{e^z}{\left(z^2+\pi^2\right)^2}dz$ where C is $|z|=4$ (6)

b) Find Fourier expansion of $f(x)=x+x^2$ in $(-\pi,\pi)$ and $f(x+2\pi)=f(x)$ (6)

c) Find i) $L\left(e^{-4t}\int_0^t u\sin 3u\,du\right)$ ii) $L^{-1}\left(\dfrac{1}{s}\log\left(1+\dfrac{1}{s^2}\right)\right)$ (8)

6 a) Show that the function $w=\dfrac{4}{z}$ transform the straight lines x = c in the z-plane into circles in the w-plane. (6)

b) Solve using Laplace Transform $R\dfrac{dQ}{dt}+\dfrac{Q}{c}=V$, Q = 0 when t = 0 (6)

c) Solve the Laplace equation $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$ for the following data by successive iterations (Calculate first two iterations) (8)

0	8.7	12.1	12.8	9.0
0	u_7	u_8	u_9	17.0
0	u_4	u_5	u_6	21.0
0	u_1	u_2	u_3	21.9
0	11.1	17.0	19.7	18.6