

Time Duration: 3Hr

Total Marks: 80



N.B.:1) Question no.1 is compulsory.

2) Attempt any three questions from Q.2to Q.6.

3) Use of statistical tables permitted.

4) Figures to the right indicate full marks.

Maximum

Marks

- Q1. a) Evaluate  $\int_C |z| dz$ , where C is the left half of unit circle  $|z| = 1$  from  $z = -i$  to  $z = i$ . [5]
- b) If  $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ , then find the eigen values of  $4A^{-1} + 3A + 2I$ . [5]
- c) If the tangent of the angle made by the line of regression of y on x is 0.6 and  $\sigma_y = 2\sigma_x$ , find the correlation coefficient between x and y. [5]
- d) Construct the dual of the following L.P.P. [5]

$$\begin{aligned} \text{Minimise } z &= x_2 + 3x_3 \\ \text{Subject to } 2x_1 + x_2 &\leq 3 \\ x_1 + 2x_2 + 6x_3 &\geq 5 \\ -x_1 + x_2 + 2x_3 &= 2 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

- Q2. a) Evaluate  $\int_C \frac{e^{2z}}{(z+1)^4} dz$ , where c is the circle  $|z - 1| = 3$ . [6]
- b) Show that the matrix  $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$  is derogatory. [6]
- c) For a normal variate with mean 2.5 and standard deviation 3.5, find the probability that (i)  $2 \leq X \leq 4.5$ , (ii)  $-1.5 \leq X \leq 5.3$ . [8]
- Q3. a) The daily consumption of electric power is a random variable X with probability [6]

$$\text{distribution function } f(x) = \begin{cases} kxe^{-\frac{x}{3}}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Find the value of k, the expectation of k and the probability that on a given day the electric consumption is more than expected value.

- b) Solve the following L.P.P. by simplex method [6]
- $$\begin{aligned} \text{Maximise } z &= 4x_1 + 10x_2 \\ \text{Subject to } 2x_1 + x_2 &\leq 10 \\ 2x_1 + 5x_2 &\leq 20 \\ 2x_1 + 3x_2 &\leq 18 \\ x_1, x_2 &\geq 0 \end{aligned}$$
- c) Expand  $f(z) = \frac{2}{(z-1)(z-2)}$  in the regions (i)  $|z| < 1$  (ii)  $1 < |z| < 2$  (iii)  $|z| > 2$ . [8]

- Q4. a) The incidence of an occupational disease in an industry is such that the workers have 20% chance of suffering from it. What is the probability that out of 6 workers chosen at random 4 or more will be suffering from the disease? [6]
- b) Calculate the coefficient of correlation between X and Y from the following data. [6]

X	3	5	4	6	2
Y	3	4	5	2	6

- c) Show that the matrix  $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$  is diagonalizable. Find the transforming matrix M and the diagonal form D. [8]

Q5.a) Can it be concluded that the average life- span of an Indian is more than 70 years, if a random sample of 100 Indians has an average life span of 71.8 years with standard deviation 8.9 years? [6]

b) Evaluate  $\int_0^{2\pi} \frac{d\theta}{3+2\cos\theta}$ , using Cauchy's residue theorem. [6]

c) Using the Kuhn – Tucker conditions, solve the following N.L.P.P. [8]

Maximise  $z = x_1^2 + x_2^2$   
 Subject to  $x_1 + x_2 - 4 \leq 0$   
 $2x_1 + x_2 - 5 \leq 0$   
 $x_1, x_2 \geq 0$

Q6.a) A die was thrown 132 times and the following frequencies were observed. [6]

No obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased.

b) Two independent samples of sizes 8 and 7 gave the following results. [6]

Sample 1	19	17	15	21	16	18	16	14
Sample 2	15	14	15	19	15	18	16	

Is the difference between sample means significant?

b) Using Penalty (Big-M) method solve the following L.P.P. [8]

Maximise  $z = 3x_1 - x_2$   
 Subject to  $2x_1 + x_2 \leq 2$   
 $x_1 + 3x_2 \geq 3$   
 $x_2 \leq 4$   
 $x_1, x_2 \geq 0$

ALL THE BEST!