Q.P. Code :23022

[Time: Three Hours]   [Marks:80]

Please check whether you have got the right question paper.

N.B: 1. Question.No.1 is compulsory.
        2. Attempt any three questions from Q.2 to Q.6
        3. Use of statistical table permitted.
        4. Figures to the right indicate full marks.

Q.1   a) Evaluate $\int_c \log z\, dz$ where C is the unit circle in the z – plane.   05

        b) Find the eigen values of the adjoint of $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ 05

        c) If the arithmetic mean of regression coefficient is p and their difference is 2q, find the correlation coefficient.   05

        d) Write the dual of the following L.P.P.   05
            Maximise $Z = 2x_1-x_2+4x_3$
            Subject to $x_1+2x_2-x_3 \leq 5$
                          $2x_1-x_2+x_3 \leq 6$
                          $x_1+x_2+3x_3 \leq 10$
                          $4x_1+x_3 \leq 12$
                          $x_1,x_2,x_3 \geq 0$

Q.2   a) Evaluate $\int_C \frac{\cot z}{z} dz$ where C is the ellipse $9x^2 + 4y^2 = 1$ 06

        b) Show that $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ is non- derogatory.   06

        c) If X is a normal variate with mean 10 and standard deviation 4, find   08
            i)$P(|X - 14|<1)$, ii) $P(5\leq X \leq 18)$, iii) $P(X\leq 12)$

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Q.3 a) Find the expectation of number of failures preceeding the first success in an infinite series of independent trials with constant probabilities p & q of success and failure respectively. 06

b) Using Simplex Method solve the following L.P.P
Maximise $Z = 10x_1 + x_2 + x_3$
Subject to $x_1 + x_2 - 3x_3 \leq 10$ 06
$4x_1 + x_2 + x_3 \leq 20$
$x_1, x_2, x_3 \geq 0$

c) Expand $f(z) = \dfrac{1}{z(z+1)(z-2)}$
  (i)   Within the unit circle about the origin.
  (ii) with in the annulus region between the concentric circles about the origin having 08
        radii 1 and 2 respectively.
  (iii) In the exterior of the circle with centre at the origin and radius 2.

Q.4 a) If X is Binomial distributed with mean=2 and variance = 4/3, find the probability 06
distribution of X.

b) Calculate the value of rank correlation coefficient from the following data regarding score 06
of 6 students in physics & chemistry test.
Marks in Physics   :   40, 42, 45, 35, 36, 39
Marks in Chemistry :   46, 43, 44, 39, 40, 43

c) Is the matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalisable? If so find the diagonal form and the 08
transforming matrix.

Q.5 a) A random sample of 50 items gives the mean 6.2 and standard deviation 10.24 . Can it be 06
regarded as drawn from a normal population with mean 5.4 at 5% level of significance?

b) Evaluate $\int_0^\infty \dfrac{dx}{(x^2+a^2)^3}$, a>0 Using Cauchy's residue theorem. 06

c) Using Kuhn-Tucker condition to solve the following N.L.P.P
Maximise $Z = 8x_1 + 10x_2 - x_1^2 - x_2^2$ 08
Subject to $3x_1 + 2x_2 \leq 6$
$x_1, x_2 \geq 0$

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Q.6   a) The following table gives the number of accidents in a city during a week. Find whether   06
           the accidents are uniformly distributed over a week.

Day:	Sun,	Mon,	Tue,	Wed,	Thu,	Fri,	Sat,	Total.
No. of accidents:	13	15	9	11	12	10	14	84

       b) If two independent random samples of sizes 15 & 8 have respectively the following   06
           means and population standard deviations,
           $\overline{X_1} = 980$ $\overline{X_2} = 1012$
           $\sigma_1 = 75$ $\sigma_2 = 80$
           Test the hypothesis that $\mu_1 = \mu_2$ at 5% level of significance,

           (Assume the population to be normal)

                                                                                                08
       c) Using Penally (Big M) method solve the following L.P.P.
           Minimise $Z = 2x_1 + x_2$
           Subject to $3x_1 + x_2 = 3$
                     $4x_1 + 3x_2 \geq 6$
                     $x_1 + 2x_2 \leq 3$
                     $x_1, x_2 \geq 0$

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