**(3 Hours)** **(Total Marks : 80)**

Note: 1. Question no. **1** is compulsory.

2. Attempt any **three** questions out of remaining **five** questions.

Q.1.[a] Evaluate $L[\sin 2t \cos t \cosh 2t]$. [5]

[b] How many friends must you have to guarantee that atleast five of them have [5] birthday in the same month.

[c] Determine the constants a, b, c, d, e so that the function [5]

$$f(z) = ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2 + i\left(4x^3y - exy^3 + 4xy\right) \text{ is analytic.}$$

[d] Out of one lakh people 51500 are female and 48500 are male. Among the females [5] 9000 are singers, among the males 30200 are singers. A person chosen randomly. If A, B, C are the events that a singer is chosen, a female is chosen and male is chosen respectively then find (i) P(A/B) (ii) P(A/C) (iii) P̶(̶A̶/̶C̶)̶ P(B/A) (iv) P(C/A).

Q.2. [a] Using Venn diagram show that $P \cap (Q \oplus R) = (P \cap Q) \oplus (P \cap R)$. [6]

[b] Evaluate $L\{f(t)\}$ where $f(t) = \begin{cases} 1 & 0 \le t < a \\ -1 & a < t < 2a \end{cases}$ and $f(t+2a) = f(t)$. [6]

[c] Let f, g, h be the functions shown in the diagraph : [8]
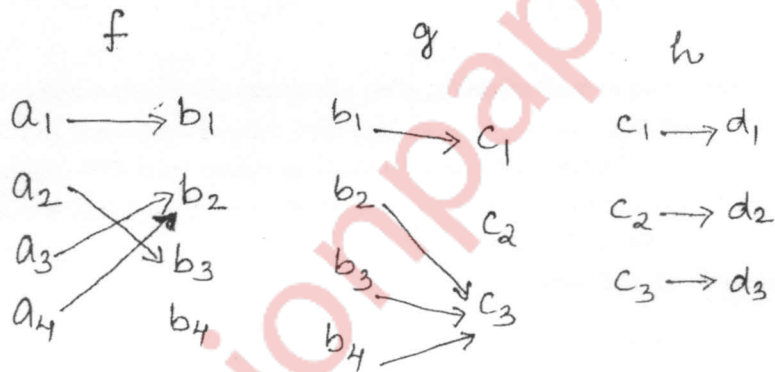
Find : (i) g∘f, h∘(g∘f), (h∘g)∘f , h⁻¹

(ii) Identify onto and one-one function for 3 of them.

Q.3. [a] Find analytic function $f(z) = u + iv$ where $v = \dfrac{x}{x^2 + y^2} + \cosh x \cos y$. [6]

[b] Solve $(D^2 + 2D + 5)y = e^{-t} \sin t$, when $y(0) = 0$, $y'(0) = 1$. [6]

[c] Evaluate (i) $L\left\{\dfrac{1}{t}(1 - \cos t)\right\}$ [8]

(ii) $\displaystyle\int_0^\infty e^{-t}\left(\int_0^t u^4 \sinh u \cosh u \; du\right) dt$

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Q.4. [a] Evaluate using convolution theorem $L^{-1}\left[\dfrac{(s+2)}{(s^2+4s+8)^2}\right]$ [6]

[b] Find bilinear transformation which maps the points $z=-1,1,\infty$ onto $w=-i,-1,i$.   [6]

[c] Three machines A, B and C produce respectively 25%, 35% and 40% of the total number of items of a factory. The percentages of defective output of these machines are respectively 5%, 4% and 2%. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine A.   [8]

Q.5. [a] Suppose repetitions are not permitted.   [6]
    (i) How many four- digit numbers can be formed from the digits 1, 2, 3, 5, 7, 8?
    (ii) How many of the numbers in part (a) are less than 4000?
    (iii) How many of the numbers in part (a) are multiples of 5?

[b] Let A={1,2,3,4,12} and let R be the relation on A defined by xRy if and only if "x divides y", Show that (A.R) is a PO set. Draw the diagraph of R.   [6]

[c] Evaluate   (i) $L^{-1}\left[\dfrac{e^{-5s}}{(s-2)^4}\right]$ (ii) $L^{-1}\left[\log\left(\dfrac{s+3}{s+5}\right)\right]$ [8]

Q.6. [a] It is known that at the university 60% of the professors play tennis, 50% of them play bridge, 70% jog, 20% play tennis and bridge, 30% play tennis and jog, 40% play bridge and jog. If someone claimed that 20% of the professors jog and play bridge and tennis, would you believe this claim? Why?   [6]

[b] Solve $a_{r+2} + 2\,a_{r-1} - 3a_r = 0$ that satisfies $a_0 = 1$, $a_1 = 2$.   [6]

[c] (i) If f(z) is an analytic and | f(z) | is constant, show that f(z) is constant.   [8]

(ii) Find the image of |z-ai| = a under the transformation $w = \dfrac{1}{z}$.

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