

Applied Maths-III



Q.P. Code : 13608

(3 Hours)

[ Total marks : 80

- Note :-
- 1) Question number 1 is compulsory.
  - 2) Attempt any **three** questions from the remaining **five** questions.
  - 3) **Figures** to the **right** indicate **full** marks.

Q 1.A) Show that  $u = y^3 - 3x^2y$  is a harmonic function. Also find its harmonic conjugate. (5)

B) Find half range Fourier sine series for  $f(x) = x^3, -\pi < x < \pi$ . (5)

C) If  $\vec{F} = xye^{2z}i + xy^2\cos zj + x^2\cos xyk$  find  $\text{div}\vec{F}$  and  $\text{curl}\vec{F}$  (5)

D) Evaluate  $\int_0^\infty e^{-2t}\sin^3 t dt$ . (5)

Q.2) A) Prove that  $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$  (6)

B) Find an analytic function  $f(z)$  whose imaginary part is  $e^{-x}(y\sin y + x\cos y)$  (6)

C) Obtain Fourier series for  $f(x) = 1 + \frac{2x}{\pi} \quad -\pi \leq x \leq 0$   
 $= 1 - \frac{2x}{\pi} \quad 0 \leq x \leq \pi$

Hence deduce that  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$  (8)

Q.3) A) Show that  $\vec{F} = (2xyz^2)i + (x^2z^2 + z\cos yz)j + (2x^2yz + y\cos yz)k$ , is a conservative field. Find its scalar potential  $\varphi$  such that  $\vec{F} = \nabla\varphi$  and hence, find the work done by  $\vec{F}$  in displacing a particle from  $A(0,0,1)$  to  $B(1,\pi/4,2)$  along straight line AB (6)

B) Show that the set of functions  $f_1(x) = 1, f_2(x) = x$  are orthogonal over  $(-1, 1)$ . Determine the constants  $a$  and  $b$  such that the function  $f_3(x) = -1 + ax + bx^2$  is orthogonal to both  $f_1$  and  $f_2$  on that interval (6)

**TURN OVER**

C) Find (i)  $L^{-1}\left\{\log\left[\frac{s^2+a^2}{\sqrt{s+b}}\right]\right\}$

(ii)  $L\{(e^{-t}\cos t.H(t-\pi))\}$  (8)

Q.4) A) Prove that  $\int J_5(x) dx = -J_4(x) - \frac{4}{x}J_3(x) - \frac{8}{x^2}J_2(x)$  (6)

B) Find inverse Laplace of  $\frac{s}{(s^2-a^2)^2}$  using Convolution theorem. (6)

C) Expand  $f(x) = \frac{3x^2-6x\pi+2\pi^2}{12}$  in the interval  $0 \leq x \leq 2\pi$  as a Fourier series.

Hence, deduce that  $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  (8)

Q.5) A) Using Gauss Divergence theorem, prove that  $\iint_S (y^2z^2i + z^2x^2j + z^2y^2k) \cdot \bar{N} ds = \frac{\pi}{12}$

where S is the part of the sphere  $x^2 + y^2 + z^2 = 1$  and above the xy-plane. (6)

B) Prove that  $J_3(x) + 3J_0(x) + 4J_0'''(x) = 0$  (6)

C) Solve  $(D^3-2D^2+5D)y = 0$ , with  $y(0)=0$ ,  $y'(0)=0$  and  $y''(0)=1$ , (8)

Q.6) A) Evaluate by Green's theorem for  $\int_C \left(\frac{1}{y} dx + \frac{1}{x} dy\right)$  where C is the

the boundary of the region define by  $x = 1$ ,  $x = 4$ ,  $y = 1$  and  $y = \sqrt{x}$  (6)

B) Find the bilinear transformation which maps the points  $z = 1, i, -1$  onto points  $w = i, 0, -i$  (6)

C) Find Fourier cosine integral representation for  $f(x) = e^{-ax}, x > 0$

Hence, show that  $\int_0^\infty \frac{\cos\omega x}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}, x \geq 0$  (8)

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