Sem-III / EXTC & INST / App. Maths-III / CBGS
18-05-16

QP Code : 30598

(Revised course)

Time :3 hours Total marks :80

N.B : (1) Question No.1 is compulsory.
(2) Answer any three questions from remaining.
(3) Assume suitable data if necessary.

1. (a) Evaluate

$$\int_0^{\infty} e^{-2t}\left(\frac{\sinh t \sin t}{t}\right) dt$$ 05

(b) Obtain the Fourier Series expression for $f(x)=9-x^2$ in (-3,3) 05

(c) Find the value of 'p' such that the function f(z) expressed in polar co-ordinates as $f(z)=r^3\cos p\theta + ir^p \sin 3\theta$ is analytic. 05

(d) If $\bar{F}=(y^2-z^2+3yz-2x)\hat{i}+(3xz+2xy)\hat{j}+(3xy-2xz+2z)\hat{k}$. Show that $\bar{F}$ is irrotational and solenoidal . 05

2. (a) Solve the differential equation using Laplace Transform 06

$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+8y=1$, given y(0)=0 and y'(0)=1

(b) Prove that 06

$$J_4(x)=\left(\frac{48}{x^3}-\frac{8}{x}\right)J_1(x)-\left(\frac{24}{x^2}-1\right)J_0(x)$$

(c) i) Find the directional derivative of $\phi=4xz^3-3x^2y^2z$ at (2,-1,2) in the direction of $2\hat{i}+3\hat{j}+6\hat{k}$. 08

ii) If $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$

Prove that $\nabla \log r=\frac{\bar{r}}{r^2}$

3. (a) Show that $\{\cos x, \cos 2x, \cos 3x........\}$ is a set of orthogonal functions over $(-\pi,\pi)$ .Hence construct an orthonormal set. 06

(b) Find an analytic function f(z) =u+iv where. 06

$$u = \frac{x}{2}\log(x^2+y^2) - y\tan^{-1}\left(\frac{y}{x}\right) + \sin x \cosh y$$

(c) Find Laplace transform of 08

i) $\int_0^t ue^{-3u}\cos^2 2u\,du$

ii) $t\sqrt{1+\sin t}$

4. (a) Find the Fourier Series for 06

$f(x) = \frac{3x^2 - 6\pi x + 2\pi^2}{12}$ in $(0,2\pi)$

Hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}.........=\frac{\pi^2}{6}$

(b) Prove that 06

$\int_0^b xJ_0(ax)dx = \frac{b}{a}J_1(ab)$

c) Find 08

i) $L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$

ii) $L^{-1}\left[\left(\frac{s+2}{s^2-2s+17}\right)\right]$

5. (a) Obtain the half range cosine series for 06

$$f(x) = x, 0 < x < \frac{\pi}{2}$$
$$= \pi - x, \frac{\pi}{2} < x < \pi$$

(b) Find the Bi- linear Transformation which maps the points 1,i,-1 of z plane onto i,0,-i of w-plane 06

(c) Verify Green's Theorem for $\int_C \overline{F}.\overline{dr}$ where 08

$\overline{F} = (x^2 - xy)\hat{i} + (x^2 - y^2)\hat{j}$ and C is the curve bounded by $x^2 = 2y$ and $x = y$

6.(a) Show that the transformation 06

$w = \frac{i - iz}{1 + z}$ maps the unit circle $|z| = 1$ into real axis of w plane.

(b) Using Convolution theorem find 06

$$L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]$$

(c) 08

i) Use Gauss Divergence Theorem to evaluate $\iint_S \overline{F}.\hat{n}ds$ where $\overline{F} = x\hat{i} + y\hat{j} + z\hat{k}$ and S is the sphere $x^2 + y^2 + z^2 = 9$ and $\hat{n}$ is the outward normal to S

ii) Use Stoke's Theorem to evaluate $\int_C \overline{F}.\overline{dr}$ where $\overline{F} = x^2\hat{i} - xy\hat{j}$ and C is the square in the plane z=0 and bounded by x=0,y=0,x=a and y=a.