

(3 Hours)

[Total Marks : 80

- N. B. : (1) Question No. 1 (one) is **compulsory**.
 (2) Attempt any 3 (three) questions from the remaining questions.
 (3) Assume suitable data, if necessary.

1. (a) Evaluate $\int_0^{\infty} \frac{(\cos 6t - \cos 4t)}{t} dt$ 5
 (b) Obtain complex form of fourier series for $f(x) = e^{ax}$ in $(-1,1)$ 5
 (c) Find the work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. 5
 (d) Find the orthogonal trajectory of the curves $3x^2y + 2x^2 - y^3 - 2y^2 = \alpha$, where α is a constant. 5
2. (a) Evaluate $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t, y(0) = 0, y'(0) = 0$, by Laplace transform 6
 (b) Show that $J_{5/2} = \sqrt{\frac{2}{\pi x}} \left[\frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right]$ 6
 (c) (i) Find the constants a, b, c so that $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + (y + 2z))\hat{k}$ is irrotational. 4
 (ii) Prove that the angle between two surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$ is $\cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$ 4
3. (a) Obtain the fourier series of $f(x)$ given by $f(x) = \begin{cases} 0 & -\pi \leq x \leq 0 \\ x^2 & 0 \leq x \leq \pi \end{cases}$ 6
 (b) Find the analytic function $f(z) = u + iv$ where $u = r^2 \cos 2\theta - r \cos \theta + 2$ 6
 (c) Find Laplace transform of
 (i) $te^{-3t} \cos 2t \cdot \cos 3t$ 8
 (ii) $\frac{d}{dt} \left[\frac{\sin 3t}{t} \right]$

4. (a) Evaluate $\int J_3(x) dx$ and Express the result in terms of J_0 and J_1 6
 (b) Find half range sine series for 6
 $f(x) = \pi x - x^2$ in $(0, \pi)$
 Hence deduce that $\frac{\pi^3}{32} = \frac{1}{12} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \dots$
- (c) Find inverse Laplace transform of 8
 (i) $\frac{1}{s} \tanh^{-1}(s)$ (ii) $\frac{se^{-2s}}{(s^2 + 2s + 2)}$
5. (a) Under the transformation $w + 2i = z + \frac{1}{z}$, show that the map of the circle $|z| = 2$ is an ellipse in w -plane. 6
 (b) Find half range cosine series of $f(x) = \sin x$ in $0 \leq x \leq \pi$. 6
 Hence deduce that
 $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots = \frac{1}{2}$
- (c) Verify Green's theorem, for 8
 $\oint_c (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where c is boundary of the region defined by $x=0, y=0$, and $x+y = 1$.
6. (a) Using convolution theorem; evaluate 6
 $L^{-1}\left\{\frac{1}{(s-1)(s^2+4)}\right\}$
- (b) Find the bilinear transformation which maps the points 6
 $z = 1, i, -1$ onto $w = 0, 1, \infty$
- (c) By using the appropriate theorem, Evaluate the following :- 8
 (i) $\int \bar{F} \cdot d\bar{r}$ where $\bar{F} = (2x - y)\hat{i} - (yz^2)\hat{j} - (y^2z)\hat{k}$
 and c is the boundary of the upper half of the sphere $x^2 + y^2 + z^2 = 4$
 (ii) $\iiint_s (9x\hat{i} + 6y\hat{j} - 10z\hat{k}) \cdot d\bar{s}$ where s is
 the surface of sphere with radius 2 units.