SE-III | App. Maths - III | EXTC & INST

26-11-14

QP Code : 14571

(3 Hours) **[Total Marks : 80**

N.B. (1) Question No.1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Figures to right indicate full marks.

1. (a) Prove that $f(z)=x^2-y^2+2ixy$ is analytic and find $f'(z)$ 5

(b) Find the Fourier series expansion for $f(x)=|x|, \ in \ (-\pi,\pi)$ 5

(c) Using laplace transform solve the following differential equation with given condition $\frac{d^2y}{dt^2}+y=t, \ given \ that \ y(0)=1 \ \& \ y'(0)=0$ 5

(d) If $\overline{A}=\nabla(xy+yz+zx), \ find \ \nabla\cdot\overline{A} \ and \ \nabla\times\overline{A}$ 5

2. (a) If $L[J_0(t)]=\frac{1}{\sqrt{s^2+1}}$, prove that $\int_0^\infty e^{-6t}t\,J_0(4t)\,dt=3/500$ 6

(b) Find the directional derivative of $\phi=x^4+y^4+z^4 \ at \ A(1,-2,1)$ in the direction of AB where $B \ is \ (2,6,-1)$. Also find the maximum directional derivative of ϕ at $(1,-2,1)$. 6

(c) Find the Fourier series expansion for $f(x)=4-x^2, \ in \ (0,2)$ Hence deduce that $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}\cdots\cdots$ 8

3. (a) Prove that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ 6

(b) Using Green's theorem evaluate $\int_C (2x^2-y^2)dx+(x^2+y^2)dy$ where 'c' is the boundary of the surface enclosed by the lines $x=0, \ y=0, \ x=2, \ y=2$. 6

(c) i) Find Laplace Transform of $e^{-3t}\int_0^t u\sin 3u\,du$

ii) Find the Laplace transform of $\frac{d}{dt}\left(\frac{1-\cos 2t}{t}\right)$ 8

4. (a) Obtain complex form of Fourier series for the functions $f(x)=\sin ax \ \ in \ \ (-\pi,\pi)$, where a is not an integer. 6

(b) Find the analytic function whose imaginary part is $v=\frac{x}{x^2+y^2}+\cosh y\cdot\cos x$ 6

(c) Find inverse Laplace Transform of following

i) $\log\left(\frac{s^2+a^2}{\sqrt{s+b}}\right)$ ii) $\frac{1}{s^3(s-1)}$ 8

5. (a) Obtain half-range cosine series for $f(x)=x(2-x) \ in \ 0<x<2$ 6

(b) Prove that $\overline{F}=\frac{\overline{r}}{r^3}$ is both irrotational and solenoidal 6

(c) Show that the function $u=\sin x\cosh y+2\cos x\sinh y+x^2-y^2+4xy$ satisfies

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Laplace's equation and find it corresponding analytic function

6. (a) Evaluate by Stoke's theorem $\int_C (x\,y\,dx + x\,y^2\,dy)$ where C is the square in the xy-plane with vertices $(1,0), (0,1), (-1,0), and\ (0,-1)$ 6

(b) Find the bilinear transformation, which maps the points $z = -1, 1, \infty$ onto the points $w = -i, -1, i$. 6

(c) Show that the general solution of $\frac{d^2y}{dx^2} + 4x^2y = 0$ is $y = \sqrt{x}\left[A\,J_{1/4}(x^2) + B\,J_{-1/4}(x^2)\right]$ where A and B are constants 8

Course : Prog. 616 to 630 S.E. (SEM III) (CBSGS) (Even)

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Correction :

Q.3. (c) (i) Find Laplace Transform of

$$e^{-3t}\int_0^t u \sin 3u \, du$$

Query Update time : 26/11/2014