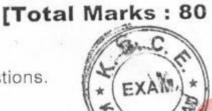
26-11-14

QP Code: 14571

(3 Hours)



PUAT, RP

- N.B. (1) Question No.1 is compulsory.
 - (2) Attempt any three questions out of the remaining five questions.
 - (3) Figures to right indicate full marks.
- 1. (a) Prove that $f(z) = x^2 y^2 + 2ixy$ is analytic and find f'(z)
 - (b) Find the Fourier series expansion for f(x) = [x], in $(-\pi, \pi)$
 - (c) Using laplace transform solve the following differential equation with given condition $\frac{d^2y}{dt^2} + y = t$, given that y(0) = 1 & y'(0) = 0
 - (d) If $\overline{A} = \nabla(xy + yz + zx)$, find $\nabla \cdot \overline{A}$ and $\nabla \times \overline{A}$
- 2. (a) If $L[J_0(t)] = \frac{1}{\sqrt{s^2 + 1}}$, prove that $\int_0^\infty e^{-6t} t J_0(4t) dt = 3/500$
 - (b) Find the directional derivative of $\phi = x^4 + y^4 + z^4$ at A(1, -2, 1) in the direction of AB where B is (2, 6, -1). Also find the maximum directional derivative of ϕ at (1, -2, 1).
 - (c) Find the Fourier series expansion for $f(x) = 4 x^2$, in(0, 2)Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$
- 3. (a) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$
 - (b) Using Green's theorem evaluate $\int_{C} (2x^2 y^2) dx + (x^2 + y^2) dy$ where 'c' is the boundary of the surface enclosed by the lines x = 0, y = 0, x = 2, y = 2.
 - (c) i) Find Laplace Transferm of $e^{-3t} \int_{0}^{t} u \sin 3u \ du$
 - ii) Find the Laplace transform of $\frac{d}{dt} \left(\frac{1 \cos 2t}{t} \right)$
- 4. (a) Obtain complex form of Fourier series for the functions $f(x) = \sin ax$ in $(-\pi, \pi)$, where a is not an integer.
 - (b) Find the analytic function whose imaginary part is $v = \frac{x}{x^2 + y^2} + \cosh y \cdot \cos x$
 - (c) Find inverse Laplace Transform of following
 - i) $\log \left(\frac{s^2 + a^2}{\sqrt{s + b}} \right)$ ii) $\frac{1}{s^3 (s 1)}$
- 5. (a) Obtain half-range cosine series for f(x) = x(2-x) in 0 < x < 2
 - (b) Prove that $\overline{F} = \frac{\overline{r}}{r^3}$ is both irrotational and solenoidal
 - (c) Show that the function $u = \sin x \cosh y + 2 \cos x \sinh y + x^2 y^2 + 4xy$ satisfies

[TURN OVER

Laplace's equation and find it corresponding analytic function

Evaluate by Stoke's theorem $\int (xydx + xy^2dy)$ where C is the square in the xy-(a)

plane with vertices (1,0), (0,1), (-1,0), and (0,-1)

- Find the bilinear transformation, which maps the points $z = -1, 1, \infty$ onto the points (b) w=-i,-1,i.
- Show that the general solution of $\frac{d^2y}{dx^2} + 4x^2y = 0$ is (c)

 $y=\sqrt{x}\left[A\ J_{1/4}\!\left(x^2\right)\!+B\ J_{-1/4}\!\left(x^2\right)\!\right]$ where A and B are constants

EXAM *

Course : Prog. 616 to 630 S.E. (SEM III) (CBSGS) (Even)

Q.P Code : 14571

Correction :

Q.3. (c) (i) Find Laplace Transform of

$$e^{-3t} \int_{0}^{t} u \sin 3u \ du$$

Query Update time : 26/11/2014