

Duration : 3 Hours

N.B . 1) Question No. 1 is compulsory .

2) Attempt any three questions out of the remaining five questions .

3) Figures to the right indicate full marks .

1. (a) Find the Laplace transform of $t^n(1 + \cosh 9t. \sinh 7t)$. 5
 (b) Find the Fourier series for $f(x) = x^2$ in $(-\pi, \pi)$ 5
 (c) Show that the vector $\vec{F} = \frac{-yi + xj}{x^2 + y^2}$ is irrotational. 5
 (d) Determine constant a,b,c,d if $f(z) = (4x^2 + ay^2 + 8bxy) + i(2cxy + 4dx^2 + 2x^2)$ is analytic. 5

2. (a) Find Fourier transform of $f(x) = \begin{cases} e^{iwx} & a < x < b \\ 0 & x < a \\ 0 & x > b \end{cases}$ 6
 (b) Solve using Laplace transform $(D^3 - 3D - 2)y = 540.t^3.e^{3t}$
 $y(0) = 0, y'(0) = 0, y''(0) = 0$. 6
 (c) Find half range cosine series for $\cos \alpha x$ in $(0, \pi)$, where α is not an integer and hence show that $\sum_{n=1}^{\infty} \frac{1}{\alpha^2 - n^2} = \frac{\alpha \pi \cot \alpha \pi}{1 + 2\alpha^2}$. 8

3. (a) If $u = (x^2 + y^2 + z^2)$ Prove that $\text{Curl}(\text{grad } u) = \vec{0}$. 6
 (b) Find Fourier Series for $f(x) = x^2 + 2x$ in $(0, 2\pi)$. 6
 (c) Evaluate $\int_0^{\infty} e^{-3t} \int_0^t (u \sinh^2 u)^2. \cosh 5u e^{3u} du. dt$ 8

4. (a) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. 6
 (b) By using Stoke's theorem evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (2x + y)i - 4z^2j - y^2zk$ and C is the boundary of the hemisphere $x^2 + y^2 + z^2 = a^2, z = 0$. 6
 (c) Find Inverse Laplace transform
 i) $\left\{ \frac{5s+3}{s^2+6s+25} \right\}$ 8
 ii) $\log \left\{ \frac{s^2+16}{s^2+81} \right\}$

TURN OVER

S.E. (sem-III) (Instu.) [choice Base] Q.P. Code: 25475
08/05/2019

5. a) Define Orthogonal set of functions on (a,b), Show that the functions $f_1(x) = 1$, $f_2(x) = 3x$ are orthogonal on (-2,2). Determine the constants P, Q such that $f_3(x) = Px^2 + Qx + 9$ is orthogonal to both $f_1(x)$ & $f_2(x)$ on the same interval . 6
- (b) Find the analytic function $f(z) = u + iv$ in terms of Z if $3u - 7v = x^3 + x^2 - 3xy^2 - y^2 - 3yx^2 + y^3 - 2xy$. 6
- (c) Verify Green's theorem for $\int_C (4xy - x^2)dx + (2x + 6y^2)dy$, C is the closed curve in the XY-plane bounded by $y = x^2$ and $x = y^2$. 8
6. (a) Find Laplace transform of $f(x) = \begin{cases} \sin 7t & 0 < t < \pi/2 \\ 2 & \pi/2 < t < \pi \end{cases}$ and $f(t) = f(t + \pi)$. 6
- (b) Find the invariant points of the Bilinear transformation $w = \frac{4z-9}{z-2}$, also express it in the normal form. 6
- (c) Obtain Complex form of Fourier series for $f(x) = \sinh x$ in $(-l, l)$ 8