Q.P. Code: 25486

Duration: 3 Hours **Max. Marks: 80**

Note: 1. Questions No. 1 is compulsory.
2. Attempt any 3 Questions from the remaining questions.
3. Figures to the right indicate full marks.

Que. 1 a. Find Laplace Transform of $e^{-4t}t\cos 3t.\sin 2t$. 5

b. Find Fourier expansion for $f(x) = x^2$ in $(-\pi, \pi)$. 5

c. Prove that $\bar{F} = \dfrac{\bar{r}}{r^3}$ is solenoidal . where $\bar{a}$ is constant vector. 5

d. Find constant a in the analytic function $\dfrac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\dfrac{ay}{x}$ 5

Que. 2 a. If f(z)=u+ iv is analytic then show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$ 6

b. By using convolution theorem, find inverse Laplace Transform of $\dfrac{s^2}{(s^2+9)(s^2+16)}$ 6

c. Find Fourier series for $f(x) = \dfrac{3x^2 - 6x\pi + 2\pi^2}{12}$ in $(0, 2\pi)$ 8

Que. 3 a. Prove that a vector field $\bar{F}$ is given by $\bar{F} = (y\sin z - \sin x)i + (x\sin z + 2yz)j + (xy\cos z + y^2)k$ is irrotational, hence find its scalar potential. 6

b. Find analytic function f(z) , whose real part is $u = \dfrac{\sin 2x}{\cosh 2y + \cos 2x}$ 6

c. By using Laplace transform, solve $y''+2y'+5y = e^{-t}\sin t$;y(0)=0,y'(0)=1 8

Que. 4 a. Find the half range Fourier cosine series for $f(x) = \begin{cases} 1 & ; 0 \le x \le 1 \\ x & ; 1 \le x \le 2 \end{cases}$ 6

b. Evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = (2x - y)i - yz^2j - y^2zk$, where is the boundary of the surface of hemisphere $x^2 + y^2 + z^2 = a^2$ lying above the xy – plane. 6

c. Find Inverse Laplace Transform of i. $\dfrac{(s+1)e^{-\pi s}}{s^2+2s+5}$ ii. $\dfrac{1}{s}\log\left(\dfrac{s+2}{s+1}\right)$ 8

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Que 5 a. Show that the set of functions { sin x , sin 3x , sin 5x ,. . . . . .} are orthogonal in $[0 , \frac{\pi}{2}]$ and find the corresponding set of orthonormal functions. 6

b. Show that the transformation $w = z^2 + z$ maps the circle $|z| = 1$ in z- plane into the cardiode $\rho = 2(1 + \cos\phi)$ in w-plane. 6

c. Verify Green's Theorem in the plane for $\oint (x^2 - y)dx + (2y^2 + x)dy$ around the boundary of the region defined by $y = x^2$ and $y = x$. 8

Que 6 a. By using Laplace transform, evaluate $\int_0^\infty e^t \frac{\sin^2 t}{t} dt$ 6

b. Find bilinear transformation which maps z=1, i, -1 into w= 0,i, -∞ and hence find the fixed points. 6

c. Using Fourier cosine integral for $f(x) = 1 - x^2, \quad 0 \le x \le 1$
$\quad = 0 \quad , \quad x > 1$ 8

Hence evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3} \right) \cos\frac{x}{2}\, dx$.

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