Paper / Subject Code: 51501 / Applied Mathematics-III

S.E. SEM III / INST / CHOICE BASED / NOV 2019 / 14.11.2019

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(3 Hours) **Total Marks: 80**

Note:- 1) Question number **1** is **compulsory**.
2) Attempt any **three** questions from the remaining **five** questions.
3) **Figures** to the **right** indicates **full** marks

Q.1
a) Evaluate Laplace transform of $t\,e^{3t}\,sin4t$ — 05
b) Find half range fourier sine series for x^2 in $(0,\pi)$ — 05
c) Find the directional derivative of $4xz^2 + x^2yz$ at (1,-2,-1) in the direction of $2\bar{\imath} - \bar{\jmath} - 2\bar{k}$ — 05
d) Find k such that $\frac{1}{2}\log(x^2 + y^2) + i\,tan^{-1}\left(\frac{kx}{y}\right)$ is analytic — 05

Q.2
a) Show that the function is Harmonic and find it's conjugate $u = e^{2x}(xcos2y - ysin2y)$ — 06
b) Evaluate $L^{-1}\left[\frac{s^2}{(s^2+9)(s^2+4)}\right]$, using convolution theorem — 06
c) Verify Green's theorem in the plane for $\int_C (xy + y^2)dx + x^2dy$, where C is the region bounded by the curves $y = x$ and $y = x^2$ — 08

Q.3
a) Solve $(D^2 + 2D + 1)y = 3te^{-t}$, $y(0) = 4$, $y'(0) = 2$ by using Laplace tranmsform. — 06
b) Show that $\bar{F} = (4xy + 3x^2z)\bar{\imath} + (2x^2 - 2z)\bar{\jmath} + (x^3 - 2y)\bar{k}$ is conservative. Find the work done in moving a particle from $A(1,0,1)$ to $B(2,1,1)$. — 06
c) Find the Fourier series for the function $f(x) = \left(\frac{\pi-x}{2}\right)^2$ in the interval $0 \le x \le 2\pi$. Hence deduce $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ — 08

Q.4
a) Obtain the Fourier Series of $xcosx$ in $(-\pi,\pi)$ — 06
b) Find the bilinear transformation which maps the points $z = i, -1, 1$ onto the points $w = 0, 1, \infty$ — 06
c) Evaluate i. $L^{-1}\left[tan^{-1}\left(\frac{a}{s}\right)\right]$ ii. $L^{-1}\left[\frac{e^{-\pi s}}{s^2-2s+2}\right]$ — 08

Q.5
a) Evaluate $\int_0^\infty e^{-t}\left[t\int_0^t e^{-4u}\,cosu\,du\right]dt$ — 06
b) Show that under the transformation $w = \frac{z-i}{z+i}$, real axis in Z-plane is mapped onto the circle $|w| = 1$ — 06
c) Find the Fourier expansion of $f(x) = x^2$ in $(0,a)$. Hence deduce that $\frac{\pi^2}{-3} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}\cdots$ — 08

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Q.6 a) Find the orthogonal trajectories of the family of curves **06**
$x^2 - y^2 + x = c$

b) Find the Fourier cosine integral representation of the function 06

$$f(x) = \begin{cases} 1 - x^2, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\frac{x}{2}\, dx$

c) Evaluate by using Gauss Divergence theorem $\iint_S \bar{N}.\bar{F}\, ds$, where $\bar{F} = 4x\bar{i} + 3y\bar{j} - 2z\bar{k}$. S is the surface bounded by $x=0$, $y=0$, $Z=0$ and $2x + 2y + z = 4$. 08

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