**Q.P. Code: 25474**

**Duration: 3 Hours** **Max. Marks: 80**

Note: 1. Questions No. 1 is compulsory.
2. Attempt any 3 Questions from the remaining questions.
3. Figures to the right indicate full marks.

Que. 1 a. Find the Laplace Transform of $e^{-4t}tcos2t.sin5t.$ 5

b. Find the Fourier expansion for $f(x) = x$ in $(-\pi, \pi)$. 5

c. Prove that $\bar{F} = \frac{\bar{a}X\bar{r}}{r^n}$ is solenoidal where $\bar{a}$ is constant vector. 5

d. Find a , b ,c, d if $f(z)=(x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$ is analytic. 5

Que. 2 a. If f(z)=u+ iv is analytic then show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)u^2 = 2|f'(z)|^2$ 6

b. By using convolution theorem, find the Inverse Laplace Transform of

$$\frac{s}{(s^2+4)(s^2+9)}$$ 6

c. Find Fourier series for $f(x) = x$ ; $0 < x < 2$ and hence deduce that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{94}$$ 8

Que. 3 a. Prove that a vector field $\bar{F}$ is given by
$\bar{F} = (ysinz - sinx)i + (xsin\, z + 2yz)j + (xycosz + y^2)k$ is irrotational, hence find its scalar potential. 6

b. Find analytic function f(z) , whose imaginary part is $v = \frac{\sinh 2y}{\cosh 2y + \cos 2x}$ 6

c. By using Laplace transform, solve $y''+25y = 10\cos 2t$ ; $y(0)=2$, $y'(0) = 0$ 8

Que. 4 a. Find half range Fourier cosine series of the function

$f(x)=Lx - x^2$ ; $0 < x < L$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, 6

b. Evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = (2x-y)i - yz^2j - y^2zk$ , where is the boundary of the surface of hemisphere $x^2 + y^2 + z^2 = a^2$ lying above xy – plane. 6

c. Find Inverse Laplace Transform of a. $\frac{e^{4-3s}}{(s+4)^{5/2}}$ b. $\tan^{-1}(s+1)$ 8

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Que 5 a. Find the complex form of Fourier series of the following functions

$f(x) = e^{ax}, -\pi < x < \pi$ 6

b. Show that under the transformation $w=\frac{1}{z}$ the circle $(x-3)^2+y^2=2$ the circle is mapped to the circle $(u-\frac{3}{7})^2+v^2=\frac{2}{49}$ 6

c. Verify Green's Theorem in the plane for $\oint(x^2-y)dx+(2y^2+x)dy$ around boundary of the region defined by $y=2x^2$ and $y=2x$ 8

Que 6 a. By using Laplace transform, evaluate $\int_0^\infty \frac{\sin 2t+\sin 3t}{te^t}dt$ 6

b. Find a bilinear transformation which maps $z=2, i, -2$ into $w=1, i, -1$ 6

c. Find the Fourier integral representation of $f(x)=\begin{cases} e^{ax}, & x\le 0 \\ e^{-ax}, & x\ge 0 \end{cases}$ 8

and hence S.T. $\int_0^\infty \frac{\cos \lambda x}{\lambda^2+a^2}d\lambda = \frac{\pi}{2a}e^{-ax}$ ; $x>0, a>0$





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