

SE - ETRX & EATC

Sem - IV - AM - IV (CBCS)

Dt: - 23/05/14

QP Code : NP-19713

(3 Hours)

[Total Marks : 80

- N.B.: (1) Questions No. 1 is compulsory.
(2) Solve any **three** from the **remaining**.

1. (a) Prove that Eigen values of a hermitian matrix are real. 5

(b) Evaluate $\oint_c \frac{e^{kz}}{z} dz$ over the circle $|z|=1$ and k is real. Hence prove 5

$$\text{that } \int_0^\pi e^{k \cos \theta} \cos(k \sin \theta) d\theta = \pi.$$

(c) Find the extremal of $\int_{x_2}^{x_1} (16y^2 - (y'')^2 + x^2) dx$ 5

(d) Find a vector orthogonal to both $u = (-6, 4, 2)$ and $v = (3, 1, 5)$. 5

2. (a) Find the curve $y = f(x)$ for which $\int_{x_1}^{x_2} y \sqrt{1+(y')^2} dx$ is minimum subject to the 6

$$\text{constraint } \int_{x_1}^{x_2} \sqrt{1+(y')^2} dx = \ell.$$

(b) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$ 6

(c) Obtain Taylor's series and two distinct Laurent's series expansion of 8

$$f(z) = \frac{z^2 - 1}{z^2 + 5z + 6} \text{ about } z = 0, \text{ indicating region of covergence.}$$

3. (a) State Cayley-Hamilton Theroern, hence deduce that $A^8 = 625I$, where 6

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

(b) Using calculus of Residues, prove that $\int_0^{2\pi} e^{\cos \theta} \cos(\sin \theta - n\theta) d\theta = \frac{2\pi}{n!}$. 6

(c) Find the plane curve of fixed perimeter and maximum area. 8

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4. (a) State Cauchy-Schwartz inequality and hence show that 6

$$\left(x^2 + y^2 + z^2\right)^{1/2} \geq \frac{1}{13} (3x + 4y + 12z), \quad x, y, z \text{ are positive.}$$

- (b) Reduce the quadratic form $Q = x^2 + y^2 - 2z^2 - 4xy - 2yz + 10xz$ to Canonical form using congruent transformation. 6

- (c) (i) If $A = \begin{bmatrix} \pi/2 & 3\pi/2 \\ \pi & \pi \end{bmatrix}$, find $\sin A$. 4

- (ii) Show that the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is Derogatory. 4

5. (a) Using Rayleigh - Ritz method, find an appropriate solution for the extremal of the 6

$$\text{functional } I[y(x)] = \int_0^1 \left[xy + \frac{1}{2}(y')^2 \right] dx \text{ subject to } y(0) = y(1) = 0.$$

- (b) Find an orthonormal basis of the following subspace of \mathbb{R}^3 , $S = \{ [1, 2, 0] [0, 3, 1] \}$. 6

- (c) Is the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalizable. If so find diagonal form and transforming matrix. 8

6. (a) Find $f(3)$, $f'(1+i)$, $f''(1-i)$, if $f(a) = \oint_c \frac{3z^2 + 11z + 7}{z-a} dz$, $c: |z|=2$. 6

- (b) Evaluate $\int_0^\infty \frac{x^3 \sin x}{(x^2 + a^2)^2} dx$ using contour integration. 6

- (c) Find the singular value decomposition of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$. 8