Q. P. Code: 36987

(3 hours) **Total Marks: 80**

N.B: (1) Question no.**1** is **compulsory.**

(2)Attempt any **three** questions from remaining **five** questions.

(3)**Figures** to the **right** indicate **full** marks.

(4)Assume suitable data if necessary.

1. (a) Find the extremal of $\int_0^1 \left(xy + y^2 - 2y^2 y'\right) dx$. **(5)**

(b) State Cauchy-Schwartz inequality in R^3 and verify it for $u = (-4, 2, 1)$ and $v = (8, -4, -2)$. **(5)**

(c) If $\lambda_1, \lambda_2, \lambda_3, ...\lambda_n$ are eigen values of A, then show that $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, ...\frac{1}{\lambda_n}$ are the eigenvalues of A^{-1} . **(5)**

(d) A random variable X has the following probability mass distribution;

X:	0	1	2
$P(X=x)$:	$3c^3$	$4c - 10c^2$	$5c - 1$

, Find c and determine $P(X < 1)$. **(5)**

2. (a) Evaluate $\int_0^{1+i} z^2\, dz$, along (i) the line $y = x$, (ii) the parabola $x = y^2$, Is the line integral independent of the path ? Explain. **(6)**

(b) A random variable X has the following density function

$$f(x) = \begin{cases} 2e^{-2x} & , x > 0 \\ 0 & , x \le 0 \end{cases}$$

, Find the m.g.f. and hence, its mean and variance. **(6)**

(c) Calculate R (Spearman's rank correlation) and r (karl-pearson's)from the following data:

X:	12	17	22	27	32
Y:	113	119	117	115	121

, Interpret your result. **(8)**

3. (a) Let $V = R^3$, Show that W is a subspace of R^3 , where $W = \{(a,b,c) : a+b+c = 0\}$, that is W consists of all vectors where the sum of their components is zero. **(6)**

(b) Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z-1| = 3$. **(6)**

(c) Show that the matrix A is diagonalizable. Also find the transforming matrix and the diagonal matrix where $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$. **(8)**

4.(a) Find the extremals of $\int_{x_0}^{x_1}\left(2xy+y'''^2\right)dx$. **(6)**

(b) A transmission channel has a per-digit error probability $p=0.01$. Calculate the probability of more than 1 error in 10 received digits using (i) Binomial and (ii) Poisson distribution. **(6)**

(c) Obtain Taylor's series and two distinct Laurent's series expansion of

$f(z)=\dfrac{z-1}{z^2-2z-3}$, indicating the region of convergence. **(8)**

5.(a) Verify the Cayley-Hamilton Theorem for matrix A and hence find A^{-1} if it exists.

where $A=\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}$ **(6)**

(b) Let R^3 have the Euclidean inner product. Use Gram-Schmidt process to transform the basis $\{u_1,u_2,u_3\}$ in to an orthonormal basis where $u_1=(1,1,1)$, $u_2=(-1,1,0)$, $u_3=(1,2,1)$ **(6)**

(c) The marks obtained by 1000 students in an examination are found to be normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be (i) between 60 and 75 (ii) more than 75. **(8)**

6. (a) Using Rayleigh-Ritz method, solve the boundary value problem using a two degree polynomial as initial solution.

$I=\int_0^1\left(2xy+y^2-y'^2\right)dx;\ 0\le X\le 1,$ given $y(0)=y(1)=0$. **(6)**

(b) Show that $A=\begin{bmatrix}4 & -2 & 2\\ 6 & -3 & 4\\ 3 & -2 & 3\end{bmatrix}$. is derogatory and find its minimal polynomial. **(6)**

(c) Using Cauchy residue theorem, evaluate the following integrals:

(i) $\int_0^{2\pi}\dfrac{d\theta}{5+3\sin\theta}$ **(4)**

(ii) $\int_{-\infty}^{\infty}\dfrac{x^2}{(x^2+a^2)(x^2+b^2)}\,dx, a>0, b>0.$ **(4)**
