

(3 Hours)

[Total Marks: 80

- N.B.:** (1) Question No.1 is compulsory.
 (2) Attempt any Three from the remaining.

1. (a) Find the extremal of the functional

$$\int_0^1 [y'^2 + 12xy] dx \text{ subject to } y(0) = 0 \text{ and } y(1) = 1.$$

- (b) Verify Cauchy - Schwartz inequality for $u = (1, 2, 1)$ and $v = (3, 0, 4)$ also find the angle between u & v . 5

- (c) If λ & X are eigen values and eigen vectors of A then prove that $\frac{1}{\lambda}$ and X are eigen values and eigen vectors of A^{-1} , provided A is non singular matrix. 5

- (d) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where $C : |z| = 2$ 5

2. (a) Find the extremal that minimises the integral 6

$$\int_{x_0}^{x_1} (16y^2 - y'^2) dx$$

- (b) Find eigen values and eigen vectors of A^3 6

where $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

- (c) Obtain Taylor's and two distinct Laurent's expansion of $f(z) = \frac{z-1}{z^2-2z-3}$ 8
 indicating the region of convergence.

[TURN OVER

- 2 -

3. (a) Verify Cayley-Hamilton Theorem for 6

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \text{ and hence find } A^{-1}$$

- (b) Using Cauchy Residue Theorem, evaluate 6

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$$

- (c) Show that a closed curve 'C' of given fixed length (perimeter) which encloses maximum area is a circle. 8

4. (a) Find an orthonormal basis for the subspace of \mathbb{R}^3 by applying Gram-Schmidt process where $S = \{(1,1,1), (0,1,1), (0,0,1)\}$. 6

- (b) Find A^{50} , where 6

$$A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$$

- (c) Reduce the following Quadratic form into canonical form & hence find its rank, index, signature and value class where, 8

$$Q = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1$$

5. (a) Using the Rayleigh-Ritz method, find an approximate solution for the 6

extremal of the functional $\int_0^1 \{xy + \frac{1}{2}y'^2\} dx$ subject to $y(0) = y(1) = 0$.

- (b) Prove that $W = \{(x,y) | x = 3y\}$ subspace of \mathbb{R}^2 . Is $W_1 = \{(a,1,1) | a \text{ in } \mathbb{R}\}$ subspace of \mathbb{R}^3 ? 6

[TURN OVER

- (c) Prove that A is diagonalizable matrix. Also find diagonal form and transforming matrix where 8

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

6. (a) By using Cauchy Residue Theorem, evaluate $\int_0^{2\pi} \frac{\cos^2 \theta}{5 + 4 \cos \theta} d\theta$. 6

- (b) Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ where $C : |z+1+i|=2$. 6

- (c) (i) Determine the function that gives shortest distance between two given points. 5

- (ii) Express any vector (a,b,c) in R^3 as a linear combination of v_1, v_2, v_3 where v_1, v_2, v_3 are in R^3 . 3
