

- N.B. (1) Question No.1 is compulsory.
 (2) Attempt any three questions out of the remaining five questions.
 (3) Figures to right indicate full marks.

- Q1. (a) Evaluate $\int_c |z| dz$, where c is the left half of unit circle $|z|=1$ from $z=-i$ to $z=i$ 5
- (b) If λ is an Eigen value of the matrix A with corresponding Eigen vector X , prove that λ^n is an Eigen value of A^n with corresponding Eigen vector X . 5
- (c) Find the extremal of $\int_{x_1}^{x_2} \frac{\sqrt{1+y'^2}}{x} dx$ 5
- (d) Find the unit vector orthogonal to both $[1,1,0]$ & $[0,1,1]$ 5
- Q2. (a) Find the curve on which the functional $\int_0^1 [y'^2 + 12xy] dx$ with $y(0)=0$ & $y(1)=1$ can be Extremised. 6
- (b) Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ 6
- (c) Obtain two distinct Laurent's series expansions of $f(z) = \frac{2z-3}{z^2-4z+3}$ in powers of $(z-4)$ indicating the region of convergence in each case 8
- Q3. (a) If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find A^{50} 6
- (b) Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where c is the circle $|z|=3$ 6
- (c) Using Rayleigh-Ritz method, find an approximate solution for the extremal of the functional $I(y) = \int_0^1 (y'^2 - 2y - 2xy) dx$ subject to $y(0)=2$, $y(1)=1$. 8

Q4. (a) Find the vector orthogonal to both $[-6, 4, 2]$ & $[3, 1, 5]$ 6

(b) Show that the matrix $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory

and find its minimal polynomial. 6

(c) Reduce the matrix of the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 + 4x_1x_3 - 2x_2x_3$ to canonical form through congruent transformation and find its rank, signature, and value class. 8

Q5. (a) Find the extremal of $\int_{x_0}^{x_1} (2xy - y''^2) dx$ 6

(b) Show that the set $W = \{[x, y, z] \mid y = x + z\}$ is a subspace of \mathbf{R}^3 under the usual addition and scalar multiplication. 6

(c) Show that the following matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is diagonalisable. Also find the

diagonal form and a diagonalising matrix. 8

Q6. (a) If $f(a) = \int_c \frac{3z^2 + 7z + 1}{z - a} dz$, where c is a circle $|z| = 2$, find the values of

i) $f(-3)$, ii) $f(i)$, iii) $f'(1-i)$ $\rightarrow f'(1-i)$ 6

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$ 6

(c) Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} and A^4 .

Where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ 8