

(3 Hours)

[Total Marks: 80]

N.B. : 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

- Q.1 (a) If  $\lambda$  is eigen value of  $A$  and  $X$  is corresponding eigen vector of  $\lambda$  then show that  $\lambda^n$  is eigen value of  $A^n$  and corresponding eigen vector is  $X$  ( $n > 0$ ). (5)
- (b) Evaluate  $\int_C \frac{z^2 - 2z + 4}{z^2 - 1} dz$ , where  $C$  is  $|z - 1| = 1$ . (5)
- (c) Find the extremals of  $\int_{x_1}^{x_2} (1 + x^2 y') y' dx$ . (5)
- (d) Find a unit vector orthogonal to both  $u = (-3, 2, 1)$  and  $v = (3, 1, 5)$ . (5)
- Q.2 (a) Find eigen values and eigen vectors of  $A^2 + 2I$  where  $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ . (6)
- (b) Find the extremals of  $\int_{x_1}^{x_2} [(y'')^2 - y^2] dx$ . (6)
- (c) Obtain Laurent's series expansion of  $f(z) = \frac{4z + 3}{z^2 - z - 6}$  at  $z = 1$ . (8)
- Q.3 (a) Using Rayleigh-Ritz method find solution for the extremal of the functional  $\int_0^1 [(y')^2 - 2y - 2xy] dx$  with  $y(0) = 2$  and  $y(1) = 1$ . (6)
- (b) Evaluate  $\int_0^{\infty} \frac{1}{(x^2 + 1)(x^2 + 9)} dx$ . (6)
- (c) Show that matrix  $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$  diagonalizable. Also find diagonal and transforming matrix. (8)

[Turnover]

- Q.4
- a) Verify Cayley Hamilton Theorem for  $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ . Also find  $A^{-1}$ . (6)
- (b) Using Cauchy's Residue Theorem evaluate  $\int_0^{2\pi} \frac{d\theta}{3 + 2\cos\theta}$ . (6)
- (c) Show that the extremal of isoperimetric problem  $I = \int_{x_1}^{x_2} (y')^2 dx$  subject to the condition  $\int_{x_1}^{x_2} y dx = k$  is a parabola. (8)
- Q.5 (a) Find  $5^A$  where  $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ . (6)
- (b) Find an orthonormal basis for the subspace of  $R^3$  by applying Gram-Schmidt process where  $S = \{(1, 1, 1), (-1, 1, 0), (1, 2, 1)\}$  (6)
- (c) Reduce the following quadratic form into canonical form and hence find its rank, index, signature and value class (8)
- $$Q = 5x_1^2 + 26x_2^2 + 10x_3^2 + 6x_1x_2 + 4x_2x_3 + 14x_3x_1.$$
- Q.6 (a) State and prove Cauchy-Schwartz inequality. Hence show that for real values of  $a, b, \theta$   $(a\cos\theta + b\sin\theta)^2 \leq a^2 + b^2$ . (6)
- (b) Show that any plane through origin is a subspace of  $R^3$ . (6)
- (c) Find the singular value decomposition of  $A = \begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix}$ . (8)