

QP Code 12440

(3 Hours)

[Total Marks : 80

N.B. : (1) Question No. 1 is compulsory.

(2) Solve any three questions from the remaining.

1. (a) Find the value of  $\mu$  which satisfy the equation. 5  
 $A^{100} X = \mu X$ , where

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & -2 \\ 1 & 1 & 0 \end{bmatrix}$$

- (b) Evaluate  $\int_0^{1+i} (x^2 + iy) dz$  along 5  
 $y = x$  and  $y = x^2$ .

- (c) Find the external of the function. 5

$$\int_{x_1}^{x_2} [y^2 - y'^2 - 2y \cosh x] dx$$

- (d) Verify Cauchy-Schwartz inequality for the vectors. 5  
 $u = (-4, 2, 1)$  &  $v = (8, -4, -2)$

2. (a) Determine the function that gives the shortest distance between two given points. 6  
 (b) Find eigen values and eigen vectors of— 6

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

- (c) Obtain Taylor's and two distinct Laurent's series expansion of  $f(z) = \frac{z-1}{z^2-2z-3}$  8  
 about  $z = 0$  indicating the region of convergence.

3. (a) Verify Cayley-Hamilton theorem for

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$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ hence find } A^{-2}.$$

- (b) Evaluate by using Residue theorem.

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$$\int_0^{2\pi} \frac{d\theta}{(2 + \cos\theta)^2}$$

- (c) Solve the boundary value problem.

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$$I = \int_0^1 \left( 2xy - y^2 - y^{1^2} \right) dx$$

given  $y(0) = y(1) = 0$  by Rayleigh-Ritz method.

4. (a) Reduce the following Quadratic form

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$$Q = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1$$

into canonical form. Hence find its rank, index and signature.

- (b) Show that the matrix  $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$  is derogatory.

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- (c) (i) Show that the set  $W = \{(1, x) \mid x \in \mathbb{R}\}$  is a subspace of  $\mathbb{R}^2$  under operations  $[1, x] + [1, y] = [1, x + y]$ ;  $k[1, x] = [1, kx]$ ;  $k$  is any scalar.  
 (ii) Is the set  $W = \{[a, i, 1] \mid a \in \mathbb{R}\}$  a subspace of  $\mathbb{R}^3$  under the usual addition and scalar multiplication?

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5. (a) Find the plane curve of fixed Perimeter and maximum area.

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- (b) Construct an orthonormal basis of  $\mathbb{R}^2$  by applying Gram Schmidt orthogonalization to  $S = \{[3, 1], [2, 2]\}$

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- (c) Show that the matrix  $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$  is diagonalizable. Also find diagonal form

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and diagonalising matrix.

6. (a) Evaluate  $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2 + 1)(x^2 + 4)} dx$  using Cauchy Residue Theorem. 6

(b) If  $\phi(\alpha) = \oint_c \frac{ze^z}{z-\alpha} dz$  where  $c$  is  $|z - 2i| = 3$  6

find  $\phi(1), \phi'(2), \phi(3), \phi'(4)$

(c) Show that the set  $V$  of positive real numbers with operations. 8

Addition :  $x + y = xy$

Scalar multiplication :  $kx = x^k$ .

is a vector space

where  $x, y$  are any two real numbers and  $k$  is any scalar.

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