

(3 Hours)

Total Marks: 80

- Note: 1) Question no 1 is compulsory.  
2) Attempt any 3 question out of remaining.  
3) Each question carries 20 Marks.  
4) Figures to right indicate full marks.

- Q.1 a) Calculate the coefficient of correlation between x and y from the following data: [5]  
 $N=10, \sum x = 140, \sum y = 150, \sum(x - 10)^2 = 180, \sum(y - 15)^2 = 215$  and  
 $\sum(x - 10)(y - 15) = 60$
- b) Evaluate  $\oint_c \log z dz$  where c is the circle with centre at origin and radius 1. [5]
- c) Find the projection of  $u = (3, 0, 4)$  along and perpendicular to  $v = (2, 3, 3)$  [5]
- d) Find the eigen values of  $3A^2 - 2A + 5I$  where  $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$  [5]
- Q.2 a) Find the extremals of  $\int_{x_1}^{x_2} (1 + x^2 y') y' dx$  [6]
- b) Using Gram-Schmidt process, transform the basis  $\{v_1, v_2, v_3\}$  into orthogonal basis [6]  
 where  $v_1 = (1, 0, 0), v_2 = (3, 7, -2), v_3 = (0, 4, 1)$ .
- c) Show that  $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & -2 \\ 0 & -6 & -3 \end{bmatrix}$  is diagonalisable and hence find the transforming matrix [8]  
 and diagonal form of A.
- Q.3 a) For a normal variable x, with mean 10 and standard deviation 4, find (i)  $P(|x-14| < 1)$  [6]  
 and (ii)  $P(x \leq 12)$
- b) Fit a binomial distribution for the following data [6]  
 $x: 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$   
 Frequency: 5 18 28 12 7 6 4
- c) Using Rayleigh-Ritz Method find the solution of  $I = \int_0^1 (2xy - y^2 - y'^2) dx$  where [8]  
 $0 \leq x \leq 1$  and  $y(0)=y(1)=0$ .
- Q.4 a) Find the lines of regression for following data [6]  
 $x: 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11$   
 $y: 11 \ 14 \ 14 \ 15 \ 12 \ 17 \ 16$
- b) If  $f(\alpha) = \oint_c \frac{3z^2 - z + 5}{z - \alpha} dz$  where C is the circle  $|z| = 3$  then find  $f(1), f(-1), f(-i)$ , [6]
- c) Check whether the set of pairs of real numbers of the form  $(1, u)$  with operations [8]  
 $(1, u) + (1, v) = (1, u + v)$  and  $k(1, u) = (1, ku)$  is a vector space.
- Q.5 a) Find the value of k such that  $f(x) = \begin{cases} k(1 - x^2) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$  is a probability function [6]  
 and hence find  $P(0.1 < x < 0.2)$  and  $P(x > 0.5)$
- b) If  $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$  then show that  $3 \cdot \tan A = A \cdot \tan 3$  [6]
- c) Find all possible expansions of  $f(z) = \frac{1}{(z-1)(z-2)}$ . [8]
- Q.6 a) Evaluate  $\int_0^{2\pi} \frac{\cos 3\theta}{5 + 4\cos\theta} d\theta$  using Cauchy Residue Theorem. [6]
- b) Show that the matrix  $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$  is non-derogatory. [6]
- c) Find the m.g.f. of Poisson's Distribution about origin. Hence find its mean and variance [8]