

Q. P. Code: 38423

(3 Hours)

Total Marks : 80

- N.B.: (1) Question no. 1 is compulsory.
 (2) Attempt any 3 questions from remaining Q. 2 to Q. 6.
 (3) Use statistical tables wherever required.
 (4) Figures to right indicate full marks.

- Q1
 a Find the coefficient of correlation from the following data: $N=10, \sum X=225$ 5
 $\sum Y=189, \sum(X-22)^2=85 \quad \sum(Y-19)^2=25$
 b Evaluate $\int_c \log z dz$ where c is $|z|=1$ 5
 c Find the projection of $u=(3,1,3)$ along and perpendicular to $v=(4,-2,2)$ 5
 d Find an eigen values of (i) $\text{Adj}(A)$ (ii) $24A^{-1}+2A-I$ Where $A=$ $\begin{pmatrix} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & -5 \\ 0 & 0 & 0 & 6 \end{pmatrix}$ 5
- Q2
 a Find the extremal of $\int_0^1 (y'^2 + x^2 - y^2) dx$ 06
 b Use Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ in to orthonormal bases where $u_1=(1,1,1), u_2=(0,1,1), u_3=(0,0,1)$ 06
 c Show that the matrix $A=$ $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ Also find diagonal and transforming matrix 08
- Q3
 a If X is a normal variable with mean 10 and standard deviation 4, Find 06
 (i) $P[|X-4| < 1]$ (ii) $p[5 < x < 18]$ (iii) $P[X < 12]$
 b Seven dice are thrown 729 times. How many times do you expect at least four dice to show 3 or 5 06
 c Using Rayleigh-Ritz method find solution for the extremal of the functional 08
 $\int_0^1 (2xy - y'^2 - y^2) dx$ given $y(0)=0$ and $y(1)=0$
- Q4
 a For the 50 students in the class mean of X is 62.4 and $16\text{Var}(X) = 9$ 06
 $\text{Var}(Y)$. Regression line of X on Y is $3Y-5X+180=0$ Find (i) Mean of Y (ii) Correlation r between X and Y (iii) Regression line of Y on X
 b Evaluate $\int_c \frac{z+1}{(z^3-2z^2)} dz$ where c is (i) $|z|=1$ (ii) $|z-2-i|=2$ (iii) $|z-1-2i|=2$ 06
 c Check whether the set of all pairs of real number of the form $(1, x)$ with operations $(1, y) + (1, x) = (1, y+x)$ and $k(1, y) = (1, ky)$ is a vector Space 08
- Q5
 a Using Cauchy residue theorem evaluate $\int_0^\infty \frac{1}{(x^2+1)(x^2+9)} dx$ 06
 b If $A=$ $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ find A^{50} 06
 c Find M.G.F. of Poisson distribution. Hence find it's mean and variance 08

Q6

a

Is the matrix A derogatory? Justify your answer where $A = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

6

b

A random variable X has the following p.d.f.

6

$f(x) = kx^2e^{-x}$ for $x > 0$ and $f(x) = 0$ otherwise. Find (i) k (ii) mean (iii) variance

(iv) M.G.F. (v) c.d.f. of X (vi) $P[0 < X < 1]$

c

Find all possible Laurent's series of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$
