

## Applied Maths-III

QP Code : 30598

( Revised course)

Time : 3 hours

Total marks : 80

- N.B : (1) Question No.1 is compulsory.  
 (2) Answer any three questions from remaining.  
 (3) Assume suitable data if necessary.

Evaluate

1. (a)  $\int_0^{\infty} e^{-2t} \left( \frac{\sinh t \sin t}{t} \right) dt$  05

(b) Obtain the Fourier Series expression for  $f(x) = 9 - x^2$  in  $(-3, 3)$  05

(c) Find the value of 'p' such that the function  $f(z)$  expressed in polar co-ordinates as  $f(z) = r^3 \cos p\theta + ir^p \sin 3\theta$  is analytic. 05

(d) If  $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ . Show that  $\vec{F}$  is irrotational and solenoidal. 05

2. (a) Solve the differential equation using Laplace Transform 06

$$\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 8y = 1, \text{ given } y(0) = 0 \text{ and } y'(0) = 1$$

(b) Prove that 06

$$J_4(x) = \left( \frac{48}{x^3} - \frac{8}{x} \right) J_1(x) - \left( \frac{24}{x^2} - 1 \right) J_0(x)$$

(c) i) Find the directional derivative of 08

$$\phi = 4xz^3 - 3x^2y^2z \text{ at } (2, -1, 2) \text{ in the direction of } 2\hat{i} + 3\hat{j} + 6\hat{k}.$$

ii) If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\text{Prove that } \nabla \log r = \frac{\vec{r}}{r^2}$$

3. (a) Show that  $\{\cos x, \cos 2x, \cos 3x, \dots\}$  is a set of orthogonal functions over  $(-\pi, \pi)$ . Hence construct an orthonormal set. 06

(b) Find an analytic function  $f(z) = u + iv$  where. 06

$$u = \frac{x}{2} \log(x^2 + y^2) - y \tan^{-1}\left(\frac{y}{x}\right) + \sin x \cosh y$$

(c) Find Laplace transform of

i)  $\int_0^1 u e^{-3u} \cos^2 2u du$

ii)  $t\sqrt{1 + \sin t}$

4. (a) Find the Fourier Series for

$$f(x) = \frac{3x^2 - 6\pi x + 2\pi^2}{12} \text{ in } (0, 2\pi)$$

Hence deduce that  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$  06

(b) Prove that

$$\int_0^b x J_0(ax) dx = \frac{b}{a} J_1(ab)$$

(c) Find

i)  $L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$

ii)  $L^{-1}\left[\frac{s+2}{s^2-2s+17}\right]$  08

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5. (a) Obtain the half range cosine series for 06

$$f(x) = x, 0 < x < \frac{\pi}{2}$$

$$= \pi - x, \frac{\pi}{2} < x < \pi$$

- (b) Find the Bi- linear Transformation which maps the points 06  
1, i, -1 of z plane onto i, 0, -i of w-plane

- (c) Verify Green's Theorem for  $\int_C \bar{F} \cdot d\bar{r}$  where 08

$$\bar{F} = (x^2 - xy)\hat{i} + (x^2 - y^2)\hat{j} \text{ and } C \text{ is the curve bounded by } x^2 = 2y$$

$$\text{and } x = y$$

- 6.(a) Show that the transformation 06

$$w = \frac{i - iz}{1 + z} \text{ maps the unit circle } |z| = 1 \text{ into real axis of } w \text{ plane.}$$

- (b) Using Convolution theorem, find 06

$$L^{-1} \left[ \frac{s}{(s^2 + 1)(s^2 + 4)} \right]$$

- (c) 08

- i) Use Gauss Divergence Theorem to evaluate  
 $\iiint_S \bar{F} \cdot \hat{n} ds$  where  $\bar{F} = x\hat{i} + y\hat{j} + z\hat{k}$  and S is the sphere  
 $x^2 + y^2 + z^2 = 9$  and  $\hat{n}$  is the outward normal to S

- ii) Use Stoke's Theorem to evaluate  $\int_C \bar{F} \cdot d\bar{r}$  where  
 $\bar{F} = x^2\hat{i} - xy\hat{j}$  and C is the square in the plane  $z=0$  and  
bounded by  $x=0, y=0, x=a$  and  $y=a$ .