

- N.B. (1) Question No.1 is compulsory.
 (2) Attempt any three questions out of the remaining five questions.
 (3) Figures to right indicate full marks.

1. (a) Prove that $f(z) = x^2 - y^2 + 2ixy$ is analytic and find $f'(z)$ 5
 (b) Find the Fourier series expansion for $f(x) = |x|$, in $(-\pi, \pi)$ 5
 (c) Using Laplace transform solve the following differential equation with given condition $\frac{d^2y}{dt^2} + y = t$, given that $y(0) = 1$ & $y'(0) = 0$ 5
 (d) If $\bar{A} = \nabla(xy + yz + zx)$, find $\nabla \cdot \bar{A}$ and $\nabla \times \bar{A}$ 5
2. (a) If $L[J_0(t)] = \frac{1}{\sqrt{s^2 + 1}}$, prove that $\int_0^\infty e^{-6t} t J_0(4t) dt = 3/500$ 6
 (b) Find the directional derivative of $\phi = x^4 + y^4 + z^4$ at $A(1, -2, 1)$ in the direction of AB where B is $(2, 6, -1)$. Also find the maximum directional derivative of ϕ at $(1, -2, 1)$. 6
 (c) Find the Fourier series expansion for $f(x) = 4 - x^2$, in $(0, 2)$
 Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ 8
3. (a) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ 6
 (b) Using Green's theorem evaluate $\int_C (2x^2 - y^2) dx + (x^2 + y^2) dy$ where 'c' is the boundary of the surface enclosed by the lines $x = 0, y = 0, x = 2, y = 2$. 6
 (c) i) Find Laplace Transform of $e^{-3t} \int_0^t u \sin 3u du$
 ii) Find the Laplace transform of $\frac{d}{dt} \left(\frac{1 - \cos 2t}{t} \right)$ 8
4. (a) Obtain complex form of Fourier series for the functions $f(x) = \sin ax$ in $(-\pi, \pi)$, where a is not an integer. 6
 (b) Find the analytic function whose imaginary part is $v = \frac{x}{x^2 + y^2} + \cosh y \cdot \cos x$ 6
 (c) Find inverse Laplace Transform of following
 i) $\log \left(\frac{s^2 + a^2}{\sqrt{s + b}} \right)$ ii) $\frac{1}{s^3(s-1)}$ 8
5. (a) Obtain half-range cosine series for $f(x) = x(2-x)$ in $0 < x < 2$ 6
 (b) Prove that $\bar{F} = \frac{\bar{r}}{r^3}$ is both irrotational and solenoidal 6
 (c) Show that the function $u = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$ satisfies

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Laplace's equation and find its corresponding analytic function

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6. (a) Evaluate by Stoke's theorem $\int_C (x y dx + x y^2 dy)$ where C is the square in the xy-

plane with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$

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- (b) Find the bilinear transformation, which maps the points $z = -1, 1, \infty$ onto the points $w = -i, -1, i$.

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- (c) Show that the general solution of $\frac{d^2 y}{dx^2} + 4x^2 y = 0$ is

$$y = \sqrt{x} \left[A J_{1/4}(x^2) + B J_{-1/4}(x^2) \right] \text{ where } A \text{ and } B \text{ are constants.}$$

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