

S.E. ETRX & EXTC sem III (CBGS) Nov-13
 sub:- AM-III 29/11/13

Ash5-D:\Data-44

Con. 7885-13.

GX-12071

(3 Hours)

[Total Marks : 80

N.B. :(1) Question no. 1 is compulsory.

(2) Attempt any three questions out of the remaining five questions.

(3) Figures to right indicate Full marks.

1. (a) Prove that real and imaginary parts of an analytic function $F(z) = u + iv$ are harmonic function. 5
- (b) Find fourier series for $f(x) = |\sin x|$ in $(-\pi, \pi)$. 5
- (c) Find the Laplace transform of $\int_0^t u e^{-3u} \sin 4u du$ 5
- (d) If $\vec{F} = xye^{2z} \hat{i} + xy^2 \cos z \hat{j} + x^2 \cos xy \hat{k}$, find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$. 5
2. (a) Using Laplace transform, solve :- 6
 $(D^2 + 3D + 2)y = e^{-2t} \sin t$ where $y(0) = 0, y'(0) = 0$.
- (b) Find the directional derivative of $d = x^2 y \cos z$ at $(1, 2, \pi/2)$ in the direction of 6
 $\vec{s} = 2\hat{i} + 3\hat{j} + 2\hat{k}$
- (c) Find the fouries series expansion for $F(x) = \sqrt{1 - \cos x}$ in $(0, 2\pi)$, Hence deduce 8
 that $\frac{1}{2} = \sum \frac{1}{4^{n^2} - 1}$.
3. (a) Prove the $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left\{ \frac{\sin x}{x} - \cos x \right\}$. 6
- (b) Evaluate by green's theorem, $\oint_C (x^2 y dx + y^3 dy)$ Where C is the closed path formed 6
 by $y = x, y = x^2$
- (c) (i) Find Laplace transform of $\frac{\cos bt - \cos at}{t}$ 4

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(ii) Find Laplace transform of :- $\frac{d}{dt} \left[\frac{\sin t}{t} \right]$

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4. (a) Show that the set of functions $\{\sin x, \sin 3x, \dots\}$ OR $\{\sin(2n+1)x : n = 0, 1, 2, \dots\}$ is orthogonal over $\left[0, \frac{\pi}{2}\right]$, Hence construct orthonormal set of functions.

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(b) Find the imaginary part whose real part is $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

(c) Find inverse Laplace transform of :-

6

(i) $\log \left(\frac{s^2 + 4}{s^2 + 9} \right)$

8

(ii) $\frac{s}{(s^2 + 4)(s^2 + 9)}$

5. (a) Obtain half range sine series for $f(x) = x^2$ in $0 < x < 3$.

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(b) A vector field \vec{F} is given by $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational and Hence find scalar point function ϕ such that $\vec{F} = \nabla\phi$

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(c) Show that the function $V = e^x (x \sin y + y \cos y)$ satisfies Laplace equation and find its corresponding analytic function and its harmonic conjugate.

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6. (a) By using stoke's theorem, evaluate $\oint_C \left[(x^2 + y^2)\hat{i} + (x^2 - y^2)\hat{j} \right] \cdot d\vec{r}$ where 'C' is the boundary of the region enclosed by circles $x^2 + y^2 = 4$, $x^2 + y^2 = 16$.

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(b) Show that under the transformation $w = \frac{5z - 4z}{4z - 2}$ the circle $|z| = 1$ in the z-plane is transformed into a circle of unity in the w-plane.

6

(c) Prove that $\int J_3(x) dx = \frac{-2J_1(x)}{x} - J_2(x)$.

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