

(3 Hours)

Total Marks :80

**Note: 1) Question No.1 is compulsory
2) Attempt any Three from the remaining**

- Q1**
- A) Find Laplace transform of $\sin\sqrt{t}$ 5
- B) Prove that $u = -r^3\sin 3\theta$ is harmonic function also find harmonic conjugate function of u 5
- C) Find a fourier series to represent $f(x) = \left(\frac{\pi-x}{2}\right)^2$ in $(0, 2\pi)$ hence deduce that $\frac{\pi^2}{6} = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$ 5
- D) Find the acute angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$ 5
- Q2**
- A) Prove that $J_{(-3/2)}(x) = -\sqrt{\frac{2}{\pi x}} \cdot \left(\frac{\cos x}{x} + \sin x\right)$ 6
- B) Find the Bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = 0, 1, \infty$ 6
- C) Obtain the fourier series for $f(x) = |x|$ in $(-\pi, \pi)$ 8
- Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1} + \frac{1}{9} + \frac{1}{25} + \dots$
- Q3**
- A) Find inverse laplace transform of (i) $2\tanh^{-1}(s)$ (ii) $e^{-4s} \cdot \frac{s}{(s+4)^3}$ 6
- B) Find the image of the rectangular region bounded by $x=0, x=3, y=0, y=2$ under the bilinear transformation $w = z + (1+i)$ 6
- C) Prove that $y = \sqrt{x} \cdot J_n(x)$ is a solution of the equation, $x^2 \frac{d^2 y}{dx^2} + (x^2 - n^2 + \frac{1}{4})y = 0$ 8
- Q4**
- A) Find Complex form of Fourier Series of $\cosh ax$ in $(-a, a)$ 6
- B) Use Gauss's Divergence theorem to evaluate $\iiint_S \vec{N} \cdot \vec{F} ds$ where $\vec{F} = 4xi + 3yj - 2zk$ and S is the surface bounded by $x=0, y=0, z=0$ and $2x + 2y + z = 4$ 6
- C) Solve using Laplace transform $(D^2 + 2D + 1)y = 3te^{-t}$, given $y(0) = 4$ and $y'(0) = 2$ 8
- Q5**
- A) Find half range cosine series for $f(x) = \begin{cases} x, & 0 < x < \left(\frac{\pi}{2}\right) \\ \pi - x, & \left(\frac{\pi}{2}\right) < x < \pi \end{cases}$ 6
- B) Find inverse Laplace transform of $\frac{1}{(s^2 + 4s + 13)^2}$ using convolution theorem 6
- C) Prove that $\vec{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k$ is a conservative field. Find (i) Scalar Potential for \vec{F} (ii) The work done in moving an object in this field from $(0, 1, -1)$ to $(\frac{\pi}{2}, -1, 2)$. 8

Q6

- A) Find the Laplace Transform of $e^{-4t} \int_0^t u \sin 3u \, du$ 6
- B) Use stoke's theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (2x-y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$ and S is the surface of hemisphere $x^2 + y^2 + z^2 = a^2$ lying above the XY- plane 6
- C) Express the function $f(x) = \begin{cases} 1 & , |x| < 1 \\ 0 & , |x| > 1 \end{cases}$ as Fourier integral .Hence evaluate $\int_0^\infty \frac{\sin w \cdot \sin wx}{w} \, dw$ 8