SE/Sem III/CBCS/ETRX/AM-III/ND-17

Q.P. Code: 25014

(3 Hours) Total Marks :80

Note: 1) Question No.1 is compulsory
2) Attempt any Three from the remaining

Q1

A) Find Laplace transform of $e^{-4t}\int_0^t u\sin 3u\,du$ — 5

B) Find the orthogonal trajectories of the curves $e^{-x}\cos y + xy = \alpha$, where α is a real constant in XY plane. — 5

C) Find a Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$ hence deduce that $\frac{\pi^2}{12} = \frac{1}{1} - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \ldots\ldots$ — 5

D) Prove that $\vec{F} = (x^2 + xy^2)\,\hat{\imath} + (y^2 + x^2y)\,\hat{\jmath}$ is irrotational and find its scalar potential — 5

Q2

A) If $u = -r^3\sin 3\theta$, find analytic function whose real part is u. — 6

B) Find the Bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$ — 6

C) Obtain the Fourier series for $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ — 8

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1} + \frac{1}{9} + \frac{1}{25} + \ldots\ldots$

Q3

A) Find inverse Laplace transform of (i) $\tan^{-1}\left(\frac{2}{s}\right)$ (ii) $e^{-4s} \cdot \frac{s}{(s+4)^3}$ — 6

B) Find Complex form of Fourier Series of $\cosh ax + \sinh ax$ in $(-a, a)$ — 6

C) Verify Greens Theorem for $\int_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ — 8

Q4

A) Prove that $\int x^4 J_1(x)\,dx = x^4 J_2(x) - 2x^3 J_3(x)$ — 6

B) Use Gauss's Divergence theorem to evaluate $\iint_S \bar{N}.\bar{F}\,ds$ where $\bar{F} = 4xi + 3yj - 2zk$ and S is the surface bounded by $x=0, y=0, z=0$ and $2x + 2y + z = 4$ — 6

C) Solve using Laplace transform $(D^2 + 2D + 1)y = 3te^{-t}$, given $y(0)=4$ and $y'(0)=2$ — 8

Q5

A Find half range cosine series for $f(x) = \begin{cases} x, & 0 < x < \left(\frac{\pi}{2}\right) \\ \pi - x, & \left(\frac{\pi}{2}\right) < x < \pi \end{cases}$ — 6

B) Find the image of real axis in z –plane onto w-plane under the bilinear transformation $w = \frac{1}{z+i}$ — 6

c) Prove that $y = \sqrt{x}\,J_n(x)$ is a solution of the equation, $x^2\frac{d^2y}{dx^2} + \left(x^2 - n^2 + \frac{1}{4}\right)y = 0$ — 8

0C4BF3468DABB1001C728E39776634B7

Q6

A) Find the constant a,b,c if the normal to the surface $ax^2 + yz + bxz^3 = c$ at P(1,2,1) is parallel to the surface $y^2 + xz = 61$ at (10,1,6) 6

B) Find inverse Laplace transform using convolution theorem $\frac{s}{(s^2+9)^2}$ 6

C) Express the function $f(x) = \begin{cases} 1 & , \quad |x| < 1 \\ 0 & , \quad |x| > 1 \end{cases}$ as Fourier integral .Hence evaluate $\int_0^\infty \frac{sinw.sinwx}{w}\, dw$ 8

0C4BF3468DABB1001C728E39776634B7