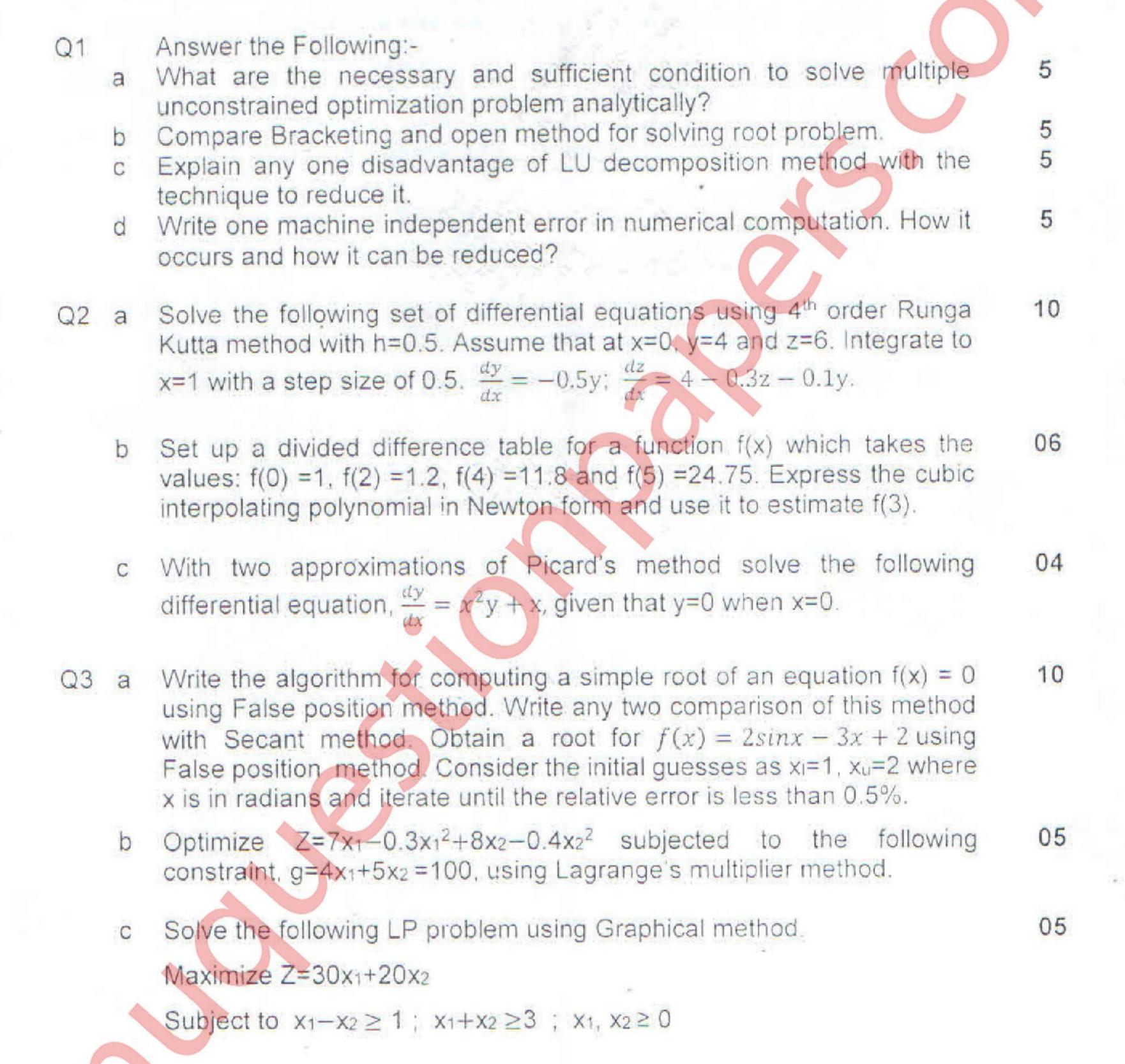(3 Hours)   [Total Marks: 80

N.B.:
- Question No. 1 is compulsory.
- Answer any **three** from the remaining five questions.
- Assume suitable data if necessary and justify the same.
- Figures to the right indicate the marks.

Q1   Answer the Following:-
   a   What are the necessary and sufficient condition to solve multiple   5
      unconstrained optimization problem analytically?
   b   Compare Bracketing and open method for solving root problem.   5
   c   Explain any one disadvantage of LU decomposition method with the   5
      technique to reduce it.
   d   Write one machine independent error in numerical computation. How it   5
      occurs and how it can be reduced?

Q2   a   Solve the following set of differential equations using 4th order Runga   10
      Kutta method with h=0.5. Assume that at x=0, y=4 and z=6. Integrate to
      x=1 with a step size of 0.5. $\frac{dy}{dx} = -0.5y$; $\frac{dz}{dx} = 4 - 0.3z - 0.1y$.

   b   Set up a divided difference table for a function f(x) which takes the   06
      values: f(0) =1, f(2) =1.2, f(4) =11.8 and f(5) =24.75. Express the cubic
      interpolating polynomial in Newton form and use it to estimate f(3).

   c   With two approximations of Picard's method solve the following   04
      differential equation, $\frac{dy}{dx} = x^2y + x$, given that y=0 when x=0.

Q3   a   Write the algorithm for computing a simple root of an equation f(x) = 0   10
      using False position method. Write any two comparison of this method
      with Secant method. Obtain a root for $f(x) = 2sinx - 3x + 2$ using
      False position method. Consider the initial guesses as x_i=1, x_u=2 where
      x is in radians and iterate until the relative error is less than 0.5%.

   b   Optimize $Z=7x_1-0.3x_1^2+8x_2-0.4x_2^2$ subjected to the following   05
      constraint, $g=4x_1+5x_2 =100$, using Lagrange's multiplier method.

   c   Solve the following LP problem using Graphical method.   05

      Maximize $Z=30x_1+20x_2$

      Subject to $x_1-x_2 \geq 1$; $x_1+x_2 \geq 3$; $x_1, x_2 \geq 0$

Q4 a What is meant by interpolation? Discuss any two advantage of Lagrange's method over Newton's divided difference. Employ inverse interpolation of order 2 to determine the value of 'x' that corresponds to $f(x)=0.93$ for the following tabulated data. Choose the sequence of the points for your estimates to attain the best possible accuracy. **10**

x	1	2	3	4
F(x)	0.5	0.8	0.9	0.941

b What is the advantage of multistep over single step method to solve ordinary differential equation? Explain with an example in each case. **10**

Q5 a Solve the equations $x^2 - y^2 = 2$ and $5x^2 - y^2 = 0$ with $x_0 = 0.4$, $y_0 = 1.0$ using N.R. method. Perform 2 iterations. **10**

b Solve the following LP problem using Simplex method. **10**

Maximize $Z = 6x_1 + 5x_2$

Subjected to $x_1 + x_2 \leq 5$; $3x_1 + 2x_2 \leq 12$; $x_1, x_2 \geq 0$.

Q6 a Employ Newton's method to find the maximum of $f(x) = 8x - x^3$. Consider the initial guess for x as 1 and iterate till the error is less than 5% **10**

b Solve the following system of equations using LU decomposition. **10**
$x + y + z = 2$
$x + 2y + 6z = 1$
$2x + 6y + z = 6$