

[Time: Three Hours]

[Marks:80]

Please check whether you have got the right question paper:

- N.B: 1. Question.No.1 is compulsory.  
 2. Attempt any three from the remaining.

Q.1. a) Find the extremal of  $\int_{x_0}^{x_1} \frac{1+y^2}{y^2} dx$  (5)

b) Is  $(6,7,-4)$  a linear combination of  $v_1 = (1,2,2), v_2 = (3,4,6)$  (5)

c) Check whether  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$  is derogatory or not. (5)

d) Evaluate  $\int_0^{1+i} z^2 dz$ , along the parabola  $x = y^2$  (5)

Q.2. a) Show that the functional  $\int_0^{\pi/2} \left\{ 2xy + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \right\} dt$ ; such that  $x(0) = 0, x\left(\frac{\pi}{2}\right) = -1,$

$y(0) = 0, y\left(\frac{\pi}{2}\right) = 1$  is stationary if  $x = -\sin t, y = \sin t$ . (6)

b) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx, a > 0, b > 0$  (6)

c) Reduce the quadratic form  $x^2 - 2y^2 + 10z^2 - 10xy + 4xz - 2zy$  to canonical form and hence find its rank, index and signature and value class. (8)

Q.3. a) Verify Cayley Hamilton theorem for  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$  and hence find  $A^{-1}$  &  $A^4$  (6)

b) Using Residue theorem evaluate  $\int_C \frac{e^z}{z^2 + \pi^2} dz$  where  $C$  is  $|z|=4$ . (6)

c) Find the singular value decomposition of  $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$  (8)

Q.4. a) If  $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ , prove that  $3 \tan A = A \tan 3$  (6)

b) Find the sum of the residues at singular points of  $f(z) = \frac{z-4}{z(z-1)(z-2)}$  (6)



c) Check whether the set of real numbers  $(x,0)$  with operation  $(x_1,0) + (x_2,0) = (x_1 + x_2,0)$ , and  $k(x_1,0) = (kx_1,0)$  is a vector space. (8)

Q.5. a) Find the extremum of  $\int_{x_0}^{x_1} (2xy - y''^2) dx$ . (6)

b) Construct an orthonormal basis of  $R^3$  using Gram Schmidt process to  $S = \{(3,0,4), (-1,0,7), (2,9,11)\}$  (6)

c) Find all possible Laurent's expansions of  $\frac{2z-3}{z^2-4z+3}$  about  $z=4$ . (8)

Q.6. a) Find the linear transformation  $Y=AX$  which carries  $X_1 = (1,1,-1)'$ ,  $X_2 = (1,-1,1)'$ ,  $X_3 = (-1,1,1)'$  onto  $Y_1 = (2,1,3)'$ ,  $Y_2 = (2,3,1)'$ ,  $Y_3 = (4,1,3)'$  (6)

b) Show that the vectors  $v_1 = (1,2,4)$ ,  $v_2 = (2,-1,3)$ ,  $v_3 = (0,1,2)$  are linearly independent. Express  $v_4 = (-3,7,2)$  in terms of  $v_1, v_2, v_3$ . (6)

c) If C is circle  $|z|=1$ , using the integral  $\int_C \frac{e^{kz}}{z} dz$  where k is real, show that  $\int_0^\pi e^{k \cos \theta} \cos(k \sin \theta) d\theta = \pi$  (8)