J -115
D -101
A -007
A -008

(180)

40 + 40        40 + 40                                    23.5.14
Electrical / EXTC / IV CBSGS
Applied Maths - IV        QP Code : **NP-19713**

(3 Hours)                                    [ Total Marks : 80

**N.B.:** (1) Questions No. **1** is **compulsory**.
(2) Solve any **three** from the **remaining**.

1. (a) Prove that Eigen values of a hermitian matrix are real.          5

(b) Evaluate $\oint_c \dfrac{e^{kz}}{z}\,dz$ over the circle $|z|=1$ and k is real. Hence prove          5

that $\displaystyle\int_0^\pi e^{k\cos\theta}\cos(k\sin\theta)\,d\theta = 2\pi$.

(c) Find the extremal of $\displaystyle\int_{x_1}^{x_2}\left(16y^2 - (y'')^2 + x^2\right)dx$          5

(d) Find a vector orthogonal to both $u = (-6, 4, 2)$ and $v = (3, 1, 5)$.          5

2. (a) Find the curve $y = f(x)$ for which $\displaystyle\int_{x_1}^{x_2} y\sqrt{1+(y')^2}\,dx$ is minimum subject to the          6

constraint $\displaystyle\int_{x_1}^{x_2}\sqrt{1+(y')^2}\,dx = \ell$.

(b) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$          6

(c) Obtain Taylor's series and two distinct Laurent's series expansion of          8

$f(z) = \dfrac{z^2 - 1}{z^2 + 5z + 6}$ about $z = 0$, indicating region of covergence.

3. (a) State Cayley-Hamilton Theroem, hence deduce that $A^8 = 625I$, where          6

$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

(b) Using calculus of Residues, prove that $\displaystyle\int_0^{2\pi} e^{\cos\theta}\cos(\sin\theta - n\theta)\,d\theta = \dfrac{2\pi}{n!}$.          6

(c) Find the plane curve of fixed perimeter and maximum area.          8

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4. (a). State Cauchy-Schwartz inequality and hence show that $\left(x^2 + y^2 + z^2\right)^{1/2} \geq \frac{1}{13}\left(3x + 4y + 12z\right)$, x, y, z are positive.    6

(b) Reduce the quadratic form $Q = x^2 + y^2 - 2z^2 - 4xy - 2yz + 10xz$ to Canonical form using congruent transformation.    6

(c) (i) If $A = \begin{bmatrix} \pi/2 & 3\pi/2 \\ \pi & \pi \end{bmatrix}$, find Sin A.    4

    (ii) Show that the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is Derogatory.    4

5. (a) Using Rayleigh - Ritz method, find an appropriate solution for the extremal of the functional $I\left[y(x)\right] = \int\limits_0^1 \left[ xy + \frac{1}{2}\left(y^1\right)^2 \right] dx$ subject to $y(0) = y(1) = 0$.    6

(b) Find an orthonormal basis of the following subspace of $R^3$, $S = \{ [1, 2, 0]\ [0, 3, 1] \}$.    6

(c) Is the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalizable. If so find diagonal form and transforming matrix.    8

6. (a) Find f(3), f'(1 + i), f''(1 – i), if $f(a) = \oint\limits_c \frac{3z^2 + 11z + 7}{z - a} dz$, $c : |z| = 2$.    6

(b) Evaluate $\int\limits_0^\infty \frac{x^3 \sin x}{\left(x^2 + a^2\right)^2}$ using contour integration.    6

(c) Find the singular value decomposition of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$.    8

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