

116

SE - Sem - IV - CBSQS - EXTC / Electrical

AM - IV

28/11/17

B - 4
18

Q. P. Code: 24361

(3 Hours)

[Total Marks: 80]

N.B. : 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) If λ is eigen value of A and X is corresponding eigen vector of λ then show that λ^n is eigen value of A^n and corresponding eigen vector is X ($n > 0$). (5)

(b) Evaluate $\int_C \frac{z^2 - 2z + 4}{z^2 - 1} dz$, where C is $|z - 1| = 1$. (5)

(c) Find the extremals of $\int_{x_1}^{x_2} (1 + x^2 y') y' dx$. (5)

(d) Find a unit vector orthogonal to both $u = (-3, 2, 1)$ and $v = (3, 1, 5)$. (5)

Q.2 (a) Find eigen values and eigen vectors of $A^2 + 2I$ where $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$. (6)

(b) Find the extremals of $\int_{x_1}^{x_2} [(y'')^2 - y^2] dx$. (6)

(c) Obtain Laurent's series expansion of $f(z) = \frac{4z + 3}{z^2 - z - 6}$ at $z = 1$. (8)

Q.3 (a) Using Rayleigh-Ritz method find solution for the extremal of the functional $\int_0^1 [(y')^2 - 2y - 2xy] dx$ with $y(0) = 2$ and $y(1) = 1$. (6)

(b) Evaluate $\int_0^{\infty} \frac{1}{(x^2 + 1)(x^2 + 9)} dx$. (6)

(c) Show that matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ diagonalizable. Also find diagonal and transforming matrix. (8)

[Turnover]

Q.4

a) Verify Cayley Hamilton Theorem for $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. Also find A^{-1} . (6)

(b) Using Cauchy's Residue Theorem evaluate $\int_0^{2\pi} \frac{d\theta}{3 + 2 \cos \theta}$. (6)

(c) Show that the extremal of isoperimetric problem $I = \int_{x_1}^{x_2} (y')^2 dx$ subject to the (8)

condition $\int_{x_1}^{x_2} y dx = k$ is a parabola.

Q.5 (a) Find S^{-1} where $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. (6)

(b) Find an orthonormal basis for the subspace of R^3 by applying Gram-Schmidt process where $S = \{(1, 1, 1), (-1, 1, 0), (1, 2, 1)\}$ (6)

(c) Reduce the following quadratic form into canonical form and hence find its rank, index, signature and value class (8)

$$Q = 5x_1^2 + 26x_2^2 + 10x_3^2 + 6x_1x_2 + 4x_2x_3 + 14x_3x_1.$$

Q.6 (a) State and prove Cauchy-Schwartz inequality. Hence show that for real values of a, b, θ $(a \cos \theta + b \sin \theta)^2 \leq a^2 + b^2$. (6)

(b) Show that any plane through origin is a subspace of R^3 . (6)

(c) Find the singular value decomposition of $A = \begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix}$. (8)