SE Electrical EXTC IV CBGS 15+9 (45)

AM - IV 01.12.16

Q. P. Code : 545800

Duration: 3 Hours (Revised Course) Total Marks: 80

N.B. : 1) Q.1. is compulsory.

2) Attempt any three from the remaining.

Q.1. a) If $f(x)$ is an algebraic polynomial in x and λ is an eigen value and X is the corresponding eigen vector of a square matrix A then $f(\lambda)$ is an eigen value and X is the corresponding eigenvector of $f(A)$. (5)

b) Find the extremal of $\int_{x_0}^{x_1}(x+y')y'dx$ (5)

c) Express $(6,11,6)$ as linear combination of $v_1=(2,1,4), v_2=(1,-1,3), v_3=(3,2,5)$. (5)

d) Evaluate $\int_C \frac{z}{(z-1)^2(z-2)}dz$, where C is the circle |z-2|=0.5 (5)

Q.2. a) Find the curve $y=f(x)$ for which $\int_0^{\pi}(y'^2-y^2)dx$ is extremum if $\int_0^{\pi} ydx=1$. (6)

b) Evaluate $\int_0^{2\pi}\frac{\cos 3\theta}{5+4\cos\theta}d\theta$ (6)

c) Find the singular value decomposition of $\begin{bmatrix}2 & 3\\ 0 & 2\end{bmatrix}$ (8)

Q.3. a) Verify Cayley Hamilton theorem for $A=\begin{bmatrix}3 & 10 & 5\\ -2 & -3 & -4\\ 3 & 5 & 7\end{bmatrix}$ and hence, find the matrix represented by $A^6-6A^5+9A^4+4A^3-12A^2+2A-I$. (6)

b) Construct an orthonormal basis of R^3 using Gram Schmidt process to S={(3,0,4),(-1,0,7),(2,9,11)} (6)

c) Find all possible Laurent's expansions of $\frac{z}{(z-1)(z-2)}$ about $z=-2$ indicating the region of convergence. (8)

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Q.4. a) Reduce the quadratic form $2x^2 - 2y^2 + 2z^2 - 2xy - 8yz + 6zx$ to canonical form and hence find its rank, index and signature and value class. (6)

b) If $\phi(\alpha) = \int_C \frac{4z^2 + z + 5}{z - \alpha} dz$, where C is the contour of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, find the values of $\phi(3.5), \phi(i), \phi'(-1), \phi''(-i)$ (6)

c) Using Rayleigh–Ritz method, solve the boundary value problem $I = \int_0^1 (y'^2 - y^2 - 2xy)dx$; $0 \le x \le 1$, given $y(0) = y(1) = 0$. (8)

Q.5. a) Find the extremal of the function $\int_0^{\pi/2} (2xy + y^2 - y'^2)dx$; with $y(0) = 0, y(\pi/2) = 0$ (6)

b) Find the orthogonal matrix P that diagonalises $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$ (6)

c) Using Cauchy's Residue theorem, evaluate $\oint_C \frac{z^2 + 3}{z^2 - 1} dz$ where C is the circle (i) $|z - 1| = 1$ (ii) $|z + 1| = 1$. (8)

Q.6. a) Find the sum of the residues at singular points of $f(z) = \frac{z}{(z-1)^2(z^2-1)}$ (6)

b) If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, prove that $A^{50} - 5A^{49} = \begin{bmatrix} 4 & -4 \\ -2 & 2 \end{bmatrix}$ (6)

c) (i) Check whether W={(x,y,z) | y=x+z, x,y,z are in R} is a subspace of R^3 with usual addition and usual multiplication. (4)

(ii) Find the unit vector in R^3 orthogonal to both u=(1,0,1) and v=(0,1,1). (4)