

SE sem-IV (choice based) Electrical
Sub - Am IV

Q.P. Code: 37579

$\frac{1}{3}$

11/05/2018

Duration - 3 Hours

Total Marks : 80

(1) N.B.: - Question no 1 is compulsory.

(2) Attempt any THREE questions out of remaining FIVE questions.

Q.1) a) Find the extremal of the functional $\int_0^{\pi/2} (y'^2 - y^2 + 2xy) dx$ with $y(0)=0$ and $y(\frac{\pi}{2})=0$. (5)

b) A continuous random variable has probability density function $f(x) = kxe^{-\frac{x}{3}}$, $x > 0$. Find 'k' and the mean. (5)

c) Find the minimal polynomial of $A = \begin{bmatrix} 2 & -3 & 3 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ and check whether it is derogatory. (5)

d) Evaluate $\int_C \frac{z^2 - 2z + 4}{z^2 - 1} dz$ where $C: |z - 1| = 1$. (5)

Q.2) a) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$, then prove that $3 \tan A = A \tan 3$. (6)

b) Evaluate $\int_C \frac{\sin \pi z + \cos \pi z}{z^2 + z} dz$; C is $|z| = 4$. (6)

c) Let $V = \{(x, y) / x, y \in \mathbb{R}, y > 0\}$. Let $(a, b), (c, d) \in V$ and $\alpha \in \mathbb{R}$. Define $(a, b) + (c, d) = (a+c, b \cdot d)$ and $\alpha(a, b) = (\alpha a, b^\alpha)$. Examine whether V is a Vector space. (8)

Q.3) a) Calculate the Karl Pearson's coefficient of correlation for the following bivariate series. (6)

| | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|----|
| X | 28 | 45 | 40 | 38 | 35 | 33 | 40 | 32 | 36 | 33 |
| Y | 23 | 34 | 33 | 34 | 30 | 26 | 28 | 31 | 36 | 35 |

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b) Find the curve $y=f(x)$ for which $\int_{x_1}^{x_2} y \sqrt{1+y'^2} dx$ is minimum
subject to the constraint $\int_{x_1}^{x_2} \sqrt{1+y'^2} dx = l$. (6)

c) Evaluate the integral $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ using Residue theorem. (8)

Q. 4 a) Let R^3 have the Euclidean inner product. Use Gram-Schmidt Process to transform the basis $\{u_1, u_2, u_3\}$ into orthonormal bases where $u_1=(1,1,1), u_2=(0,1,1), u_3=(0,0,1)$. (6)

b) Find the lines of regression for the following data (6)

| | | | | | | | | |
|---|----|----|----|----|----|----|----|----|
| x | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| y | 67 | 68 | 65 | 66 | 72 | 72 | 69 | 71 |

c) A skilled typist on routine work, kept a record of mistakes made per day during 300 working days. (8)

| | | | | | | | |
|------------------|-----|----|----|----|---|---|---|
| Mistakes per day | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| No of days | 143 | 90 | 42 | 12 | 9 | 3 | 1 |

Fit a Poisson distribution to the above data and hence calculate the theoretical frequencies.

Q. 5 a) A random variable X has the following probability distribution (6)

| | | | | |
|--------|-----|-----|-----|-----|
| X | 0 | 1 | 2 | 3 |
| P(X=x) | 1/6 | 1/3 | 1/3 | 1/6 |

Compute i) Moment generating function about the origin, ii) first two raw moments and hence the variance.

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Find eigen values and eigen vectors of $A^3 - 2A + I$ if $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ (6)

Obtain the Laurent and Taylor's series for $\frac{z-1}{z^2-2z-3}$ indicating region of convergence. (8)

Using Rayleigh-Ritz method, solve the boundary value problem $I = \int_0^1 (xy + \frac{1}{2}y'^2) dx; 0 \leq x \leq 1$ Given $y(0)=0$ and also $y(1)=0$. (6)

In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find mean and standard deviation of the distribution. (6)

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Show that for every integer $n \geq 3$, (8)

$A^n = A^{n-2} + A^2 - I$. Hence find A^{50}