

(3 Hours)

[Total marks : 80

- Note :-
- 1) Question number 1 is compulsory.
 - 2) Attempt any three questions from the remaining five questions.
 - 3) Figures to the right indicate full marks.

Q 1.A) Show that $u = y^3 - 3x^2y$ is a harmonic function. Also find its harmonic conjugate. (5)

B) Find half range Fourier sine series for $f(x) = x^3, -\pi < x < \pi$. (5)

C) If $\vec{F} = xye^{2z}i + xy^2\cos zj + x^2\cos xyk$ find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ (5)

D) Evaluate $\int_0^\infty e^{-2t} \sin^3 t dt$. (5)

Q.2) A) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ (6)

B) Find an analytic function $f(z)$ whose imaginary part is $e^{-x}(y\sin y + x\cos y)$ (6)

C) Obtain Fourier series for $f(x) = 1 + \frac{2x}{\pi} \quad -\pi \leq x \leq 0$

$$= 1 - \frac{2x}{\pi} \quad 0 \leq x \leq \pi$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ (8)

Q.3) A) Show that $\vec{F} = (2xyz^2)i + (x^2z^2 + z\cos yz)j + (2x^2yz + y\cos yz)k$, is a

conservative field. Find its scalar potential ϕ such that $\vec{F} = \nabla\phi$ and hence, find the

work done by \vec{F} in displacing a particle from $A(0,0,1)$ to $B(1,\pi/4,2)$

along straight line AB (6)

B) Show that the set of functions $f_1(x) = 1, f_2(x) = x$ are orthogonal over

$(-1, 1)$. Determine the constants a and b such that the function $f_3(x) = -1 + ax + bx^2$

is orthogonal to both f_1 and f_2 on that interval (6)

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- C) Find (i) $L^{-1}\left\{\log\left[\frac{s^2+a^2}{\sqrt{s+b}}\right]\right\}$
(ii) $L\{(e^{-t}\cos t.H(t-\pi))\}$ (8)

Q.4) A) Prove that $\int J_5(x) dx = -J_4(x) - \frac{4}{x}J_3(x) - \frac{8}{x^2}J_2(x)$ (6)

B) Find inverse Laplace of $\frac{s}{(s^2-a^2)^2}$ using Convolution theorem. (6)

C) Expand $f(x) = \frac{3x^2-6x\pi+2\pi^2}{12}$ in the interval $0 \leq x \leq 2\pi$ as a Fourier series.

Hence, deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ (8)

Q.5) A) Using Gauss Divergence theorem, prove that $\iint_S (y^2z^2i + z^2x^2j + x^2y^2k) \cdot \bar{N}ds = \frac{\pi}{12}$

where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ and above the xy-plane. (6)

B) Prove that $J_3(x) + 3J_0(x) + 4J_0'''(x) = 0$ (6)

C) Solve $(D^3-2D^2+5D)y = 0$, with $y(0)=0$, $y'(0)=0$ and $y''(0)=1$, (8)

Q.6) A) Evaluate by Green's theorem for $\int_C \left(\frac{1}{y} dx + \frac{1}{x} dy\right)$ where C is the boundary of the region defined by $x=1$, $x=4$, $y=1$ and $y=\sqrt{x}$ (6)

B) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto points $w = i, 0, -1$ (6)

C) Find Fourier cosine integral representation for $f(x) = e^{-ax}, x > 0$

Hence, show that $\int_0^\infty \frac{\cos\omega x}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}, x \geq 0$ (8)

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