(3 hours) Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Use of Statistical Tables permitted.
(4) Figures to the right indicate full marks.

Q.1 (a) Find all the basic solutions to the following problem: **05**

$$\begin{aligned} &\text{Maximise} \quad z = x_1 + 3x_2 + 3x_3 \\ &\text{subject to} \quad x_1 + 2x_2 + 3x_3 = 4 \\ &\qquad 2x_1 + 3x_2 + 5x_3 = 7 \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

(b) Evaluate $\int_c (z - z^2)\,dz$, where c is upper half of the circle $|z| = 1$. **05**

(c) Ten individual are chosen at random from a population & heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71.inches. Discuss the suggestion that the height of universe is 65 inches. **05**

(d) If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$, find A^{100} **05**

Q.2 (a) Evaluate $\int_c \dfrac{z+2}{(z-3)(z-4)}\,dz$, where c is the circle $|z| = 1$ **06**

(b) An I.Q. test was administered to 5 persons and after they were trained. The results are given below. **06**

	I	II	III	IV	V
I.Q. Before training	110	120	123	132	125
I.Q. after training	120	118	125	136	121

Test whether there is any change in I.Q. after the training programme, use 1% LOS.

(c) Solve the following LPP using Simplex Method **08**

$$\begin{aligned} &\text{Maximise} \quad z = 4x_1 + 10x_2 \\ &\text{subject to} \quad 2x_1 + x_2 \leq 10 \\ &\qquad 2x_1 + 5x_2 \leq 20 \\ &\qquad 2x_1 + 3x_2 \leq 18 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

Q.3 (a) Find the Eigen values and Eigen vectors of the following matrix. **06**

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

[Turn over

(b) If the height of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches. Find the expected number of students having heights between 65 & 71 inches. **06**

(c) Obtain Taylor's and Laurent's expansions of $f(z) = \frac{z^2-1}{z^2+5z+6}$ around $z=0$ **08**

Q.4 (a) A machine is claimed to produce nails of mean length 5 cms & standard of 0.45 cm. A random sample of 100 nails gave 5.1 as their average length. Does the performance of the machine justify the claim? Mention the level of significance you apply. **06**

(b) Using the Residue theorem, Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ **06**

(c) (i) In a certain manufacturing process 5% of the tools produced turnout to be defective. Find the probability that in a sample of 40 tools at most 2 will be defective.

(ii) A random variable x has the probability distribution **04+04**

$P(X = x_i) = \frac{1}{8}\,{}^3C_X$, $X = 0,1,2,3$. Find the moment generating function of x

Q.5 (a) Check whether the following matrix is Derogatory or Non-Derogatory: **06**

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(b) In an industry 200 workers employed for a specific job were classified according to their performance & training received to test independence of training received & performance. The data are summarized as follows. **06**

Performance	Good	Not good	Total
Trained	100	50	150
Untrained	20	30	50
Total	120	80	200

Use χ^2-test for independence at 5% level of significance & write your conclusion.

(c) Use the dual simplex method to solve the following L.P.P. **08**

Minimise $z = 2x_1 + x_2$

subject to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

[Turn over

Q.6 **(a)** Show that the matrix A satisfies Cayley-Hamilton theorem and hence find A^{-1}. **06**

Where $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

(b) A discrete random variable has the probability density function given below **06**

$X = x_i$	-2	-1	0	1	2	3
$P(x_i)$	0.2	K	0.1	2K	0.1	2K

Find K, Mean, Variance.

(c) Using Kuhn-Tucker conditions, solve the following NLPP **08**

Maximise $z = 2x_1^2 - 7x_2^2 + 12x_1x_2$

subject to $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geq 0$

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