

[Time: Three Hours]

[ Marks:80]

Please check whether you have got the right question paper.

- N.B:
1. Question no. 1 is compulsory.
  2. Attempt any three of the remaining.
  3. Figures to the right indicate full marks.



- Q.1
- a) Find the Laplace transform of  $e^{-4t} \sinh t \sin t$ . 05
  - b) Find half-range sine series for  $f(x) = \frac{\pi}{4}$  in  $(0, \pi)$ . 05
  - c) Find the values of  $Z$  for which the following function is not analytic.  
 $Z = \sin hu \cos v + i \cos hu \sin v$ . 05
  - d) Show that  $\nabla \left[ \frac{(\bar{a} \cdot \bar{r})}{r^n} \right] = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}$ , where  $\bar{a}$  is a constant vector. 05
- Q.2
- a) Find the inverse Z- transform of  $F(z) = \frac{1}{(z-3)(z-2)}$  if  $|z| < 2$ . 06
  - b) Verify Laplace's equation for  $u = \left( r + \frac{a^2}{r} \right) \cos \theta$  also find  $v$  and  $f(z)$ . 06
  - c) Find the Fourier series for the periodic function 08  

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$
 State the value of  $f(x)$  at  $x=0$  and hence, deduce that  

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$
- Q.3
- a) Find  $L^{-1} \left[ \frac{1}{(s-3)(s-3)^2} \right]$  using convolution theorem. 06
  - b) Show that the set of functions  $\sin x, \sin 2x, \sin 3x, \dots$  is orthogonal on the interval  $[0, \pi]$  06
  - c) Verify Green's Theorem for  $\int_C \bar{F} \cdot d\bar{r}$  where  $\bar{F} = x^3\mathbf{i} + xy\mathbf{j}$  and  $c$  is the triangle whose vertices are  $(0,2), (2,0)$  and  $(4,2)$ . 08

Q.4

a) Find Laplace transform of  $f(t) = \begin{cases} a \sin p t, & 0 < t < \frac{\pi}{p} \\ 0, & \frac{\pi}{p} < t < \frac{2\pi}{p} \end{cases}$  06

and  $f(t) = f\left(t + \frac{2\pi}{p}\right)$ .

b) Show that  $\vec{F} = (y^2 - z^2 + 3yz - 2x)\mathbf{i} + (3xz + 2xy)\mathbf{j} + (3xy - 2xz + 2z)\mathbf{k}$  is both solenoidal and irrotational. 06

c) Find half range cosine series for  $f(x) = x, 0 < x < 2$ . 08

Hence deduce that  $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots$

Q.5

a) Show that  $\iint_S (\nabla r^n) \cdot d\vec{s} = n(n+1) \iiint_V r^{n-2} dv$  using Gauss's Divergence theorem. 06

b) Find the Z-transform of  $\{k^2 e^{-ak}\}, k \geq 0$ . 06

c) (i) Find  $L^{-1} \left[ \frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)} \right]$  08

(ii) Find  $L^{-1} \left[ \frac{s^2 + a^2}{\sqrt{s+b}} \right]$

Q.6

a) Use Laplace transform to solve, 06

$\frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 8y = 1$  where,  $y(0) = 0, y'(0) = 1$

b) Find the bilinear transformation which maps the points  $z = \infty, i, 0$  onto the points  $0, i, \infty$  respectively of w-plane. 06

c) Express the function  $f(x) = \begin{cases} \frac{\pi}{2}, & \text{for } 0 < x < \pi \\ 0, & \text{for } x > \pi \end{cases}$  08

for Fourier Sine Integral and Show that

$$\int_0^\infty \frac{1 - \cos \pi w}{w} \sin wx \, dw = \frac{\pi}{2} \quad \text{when } 0 < x < \pi$$

\*\*\*\*\* ALL THE BEST \*\*\*\*\*