Time : 3 hrs **Marks : 80**

NB 1. Question No.I is compulsory
2. Attempt any three from the remaining six questions
3. Figures to the right indicate full marks

Q1a If Laplace transform of $erf(\sqrt{t}) = \frac{1}{s\sqrt{s+1}}$, then find $L\{e^t.erf(2\sqrt{t})\}$ [20]

b Find the Orthogonal Trajectory of the family of curves given by $e^{-x}.\cos y + x.y = c$

c Find Complex Form of Fourier Series for . e^{2x} ; $0 < x < 2$

d. If the two regression equations are $5x - 6y + 90 = 0$, $15x - 8y - 180 = 0$,

find the means of x and y, the Correlation Coefficient and Standard deviation of x if variance of Y is 1

Q2 Show that the function is Harmonic and find the Harmonic Conjugate $v = e^x.\cos y + x^3 - 3xy^2$ [6]

b Find Laplace Transform of $f(t) = \begin{cases} t & ; 0<t<1 \\ 0 & ;1<t<2 \end{cases}$, $f(t+2) = f(t)$ [6]

c. Find Fourier Series expansion of $f(x) = x - x^2, -1 < x < 1$ [8]

Q3 a Find the Analytic function $f(z) = u + iv$ if $v = \log(x^2 + y^2) + x - 2y$ [6]

b Find Inverse Z transform of $\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}$, $3 < |z| < 4$ [6]

c Solve the Differential Equation $\frac{d^2y}{dt^2} + 4y = f(t), f(t) = H(t-2), y(0) = 0, y'(0) = 1$ using Laplace Transform [8]

Q4 a Find $Z\{f(k) * g(k)\}$ if $f(k) = \left(\frac{1}{2}\right)^k, g(k) = \cos \pi k$ [6]

b Find the Spearman's Rank correlation coefficient between X and Y. [6]

X	60	30	37	30	42	37	55	45
Y	50	25	33	27	40	33	50	42

c Find the inverse Laplace transform of i) $\frac{3s+1}{(s+1)^4}$ ii) $\frac{e^{4-3s}}{(s+4)^{5/2}}$ [8]

Q5 a Find Inverse Laplace Transform usng Convolution theorem $\frac{1}{(s-4)^2(s+3)}$ [6]

b Show that the functions $f_1(x)=1$, $f_2(x)=x$ are Orthogonal on (-1,1). Determine the constants a, b such that the function $f(x)=-1+ax+bx^2$ is Orthogonal to both $f_1(x), f_2(x)$ on the (-1,1) [6]

c Find the Laplace transform of i) $e^{-3t}\int_0^t t\sin 4t\,dt$ ii) $\int_0^\infty \frac{e^{-t}-e^{-2t}}{t}dt$ [8]

Q6 a Fit a second degree parabola to the given data [6]

X	1	1.5	2	2.5	3	3.5	4
Y	1.1	1.3	1.6	2	2.7	3.4	4.1

bFind the image of $\left|z-\frac{5}{2}\right|=\frac{1}{2}$ under the transformation $w=\frac{3-z}{z-2}$ [6]

c Find Half Range Cosine Series for $f(x)=x\sin x$ in $(0,\pi)$ and hence find $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\ldots\ldots=\frac{\pi-2}{4}$ **[8]**

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