QP CODE : 40418

(3 Hours) **Max. Marks: 80**

1) Question **No.1** is **compulsory.**
2) Solve any **three** questions out of remaining **five** questions.
3) **All** questions carry **equal** marks as indicated by figures to the right.
4) Assume appropriate data whenever required. State all assumptions clearly.

Q.1 a) Use mathematical induction to show that (05M)

1+2+3...+n=n (n+1)/2 for all natural number values of n.

b) Draw Hasse Diagram for following relation, what the diagram is called as? Justify.

Let A={a,b,c,d,e} and

R={(a,a),(b,b),(c,c),(d,d),(e,e),(a,b),(b,c)(c,d),(d,e),(a,c),(a,d),(a,e),(b,d),(b,e),(c,e)} (05M)

c) Let the universal set U={1,2,3,...,10}

Let A={2,4,7,9} B={1,4,6,7,10} and C={3,5,7,9}

Find 1) AUB 2)A⋂B 3) B⋂C 4) (A⋂B)UC 5)(BUC) ⋂C (05M)

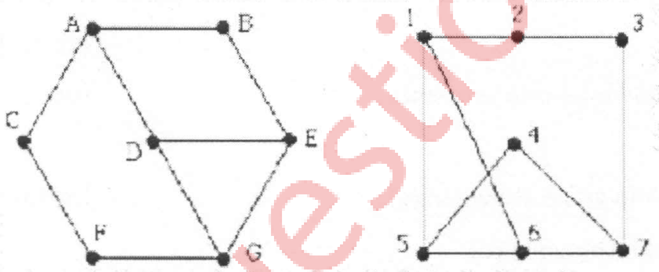
d) Consider set G = {1,2,3,4,5,6} under multiplication module 7 (05 M)

I. Find the multiplication table of the above.
II. Prove that it is a cyclic group

Q.2 a) Test whether the following function is one-to-one, onto or both. (04M)

f: Z→Z, f(x)=x^2+x+1

b) Define Isomorphic Graphs. Find if the following two graphs are isomorphic. If yes give their one-to-one correspondence. (08 M)



c) Prove that set G = {0,1,2,3,4,5} is a finite abelian group of order 6 with respect to addition modulo 6. (08M)

Q.3 a) Explain Extended Pigeonhole Principle. How many friends must you have to guarantee that at least five of them will have birthdays in the same month. (04 M)

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b) Show that the (3,6) encoding function $e:B^3 \rightarrow B^6$ defined by (08 M)

e(000)=000000 e(001)=000110

e(010)=010010 e(011)=010100

e(100)=100101 e(101)=100011

e(110)=110111 e(111)=110001 is a group code.

c) Let the functions f,g, and h defined as follows: (08 M)

$f:R \rightarrow R$, $f(x)=2x+3$

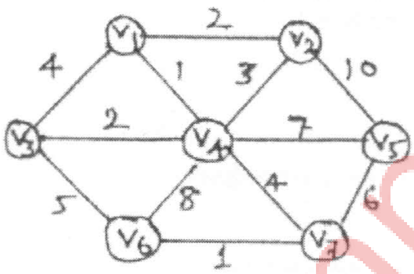
$g: R \rightarrow R$, $g(x)=3x+4$

$h: R \rightarrow R$, $h(x)=4x$

Find gof, fog, foh, , gofoh

Q.4 a) Define R on Z as aRb iff (a-b) is a non-negative even integer. Check if R is a partially ordered relation. (04 M)

b) Find Minimum spanning tree for the following graph using Prim's Algorithm. (08 M)



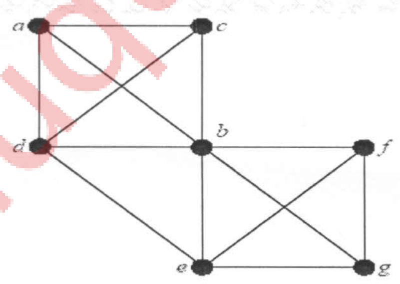
c) Solve $a_r - a_{r-1} - 6a_r = -30$ given $a_0 = 20$, $a_1 = -5$ (08 M)

Q.5 a) Find the generating function for the following finite sequences (04M)

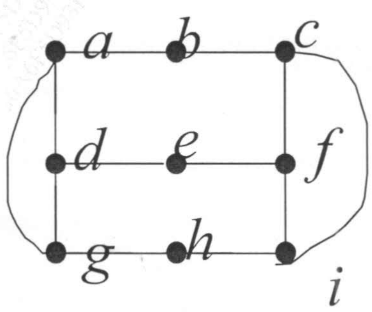
i) 2,2,2,2,2,2 ii) 1,1,1,1,1,1

b) i) Determine Hamiltonian Cycle and path in graph shown in (a) (08 M)

ii) Determine Euler Cycle and path in graph shown in (b)



(a)



(b)

c) State principle of inclusion and exclusion for three sets. A software company is looking to expand, anda firm is hired to help them find the necessary talent. The programmers must know the computer languages Java and Python. The firm receives 87 applications. Luckily, 75 applications indude knowledge of at least one of the languages. As it comes to pass, 48 applicants know Python, which is a good start, but 31 applicants do not know Java. How many people know both languages? Justify your answer with an appropriate Venn diagram. (08M)

Q.6 a) Prove p∧(q∨r) and (p∧q)∨(p∧r) are logically equivalent. (04M)

b) Let H=

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Be a parity check matrix. Determine the group code $e_H: B^3 \rightarrow B^6$ (08 M)

c) Let G be a set of rational numbers other than 1. Let * be an operation on G defined by a*b=a+b-ab for all a,b €G. Prove that (G,*) is a group. (08 M)

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