

- 1) Question no.1 is compulsory.
- 2) Solve any three questions out of remaining five questions.
- 3) All questions carry equal marks as indicated by figures to the right.
- 4) Assume appropriate data whenever required. State all assumptions clearly.

Q.1 a) Let  $A=\{a,b,c\}$ . Show that  $(P(A),\subseteq)$  is a poset. Draw its Hasse Diagram. (05M)

$P(A)$  is the power set of  $A$ .

b) Find the generating function for the following finite sequences (05M)

i) 1,2,3,4,... ii) 1,1,1,1,1,1

c) Is it possible to draw a tree with five vertices having degrees 1,1,2,2,4? (05M)

d) Prove  $p \wedge (q \vee r)$  and  $(p \wedge q) \vee (p \wedge r)$  are logically equivalent. (05M)

Q.2 a) If  $f:A \rightarrow B$  be both one-to-one and onto, then prove that  $f^{-1}: B \rightarrow A$  is also both one-to-one and onto. (04 M)

b) Let  $G$  be a set of rational numbers other than 1. Let  $*$  be an operation on  $G$  defined by  $a*b = a + b - ab$  for all  $a, b \in G$ . Prove that  $(G, *)$  is a group. (08 M)

c) Define Equivalence relation with an example. Let  $m$  be a positive integer other than 1. Show that the relation  $R = \{(a,b) | a \equiv b \pmod{m}\}$  i.e.  $m$  divides  $a-b$  is an equivalence relation on the set of integers. (08 M)

Q.3 a) Show that the set of all divisors of 70 forms a lattice. (04 M)

b). Consider the (3,5) group encoding function defined by (08 M)

$e(000) = 00000$        $e(001) = 00110$

$e(010) = 01001$        $e(011) = 01111$

$e(100) = 10011$        $e(101) = 10101$

$e(110) = 11010$        $e(111) = 11000$

Decode the following words relative to a maximum likelihood decoding function.

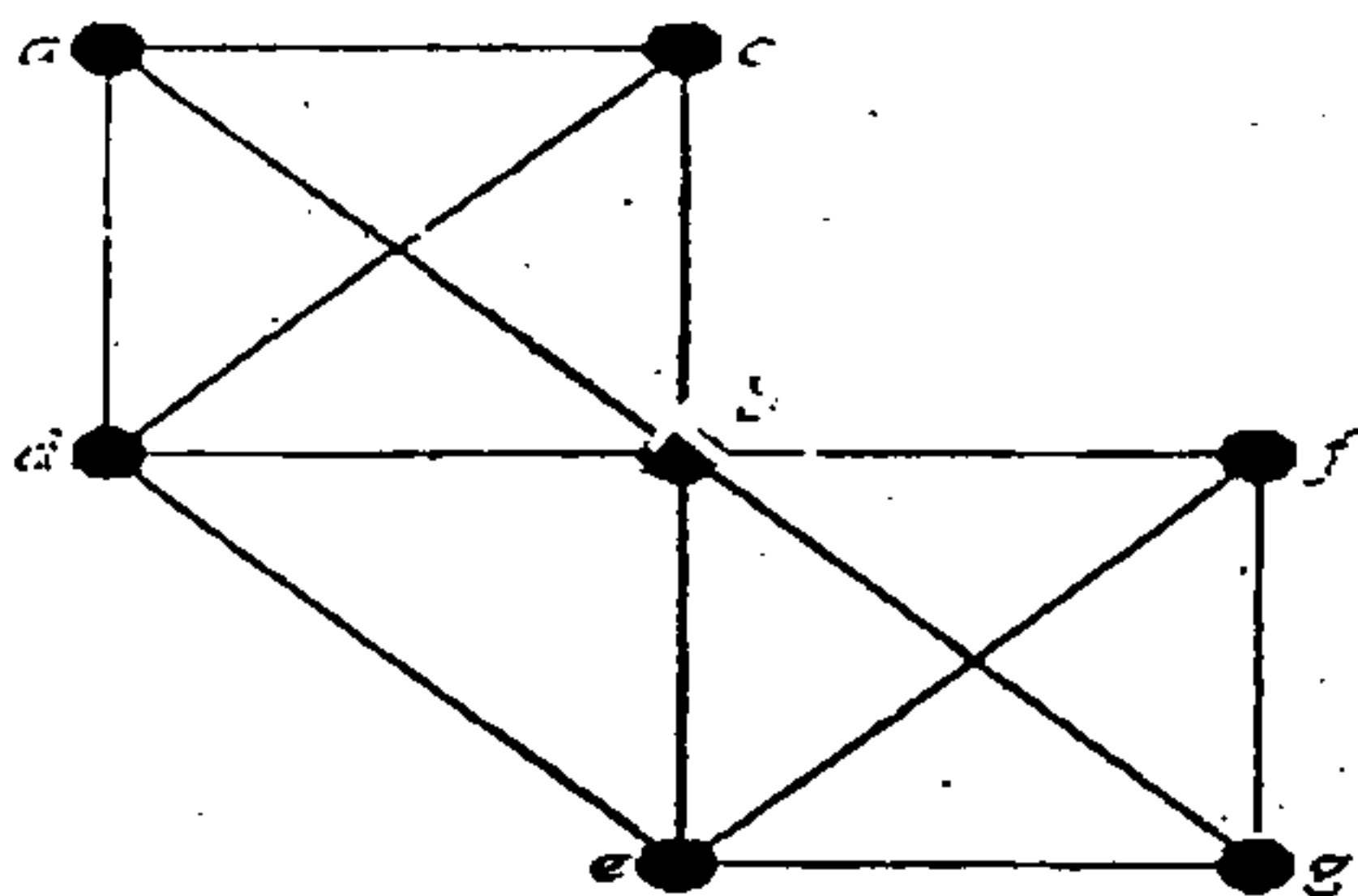
i) 11001    ii) 01010    iii) 00111

c) Define Reflexive closure, Symmetric closure along with a suitable example. Let  $R$  be a relation on set  $S = \{a,b,c,d,e\}$ , given as

$R = \{(a,a), (a,d), (b,b), (c,d), (c,e), (d,a), (e,b), (e,e)\}$

Find transitive closure using Warshall's Algorithm. (08 M)

Q.4 a) Determine Euler Cycle and path in graph shown below (04 M)



b) A survey of 500 television watchers produced the following information:  
 285 watch football games, 195 watch hockey games, 115 watch basket ball games, 45 watch football and basketball games, 70 watch football and hockey games, 50 watch basketball and hockey games. 50 do not watch any three kinds of games. Find: (08 M)

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- i) How many in the survey watch all 3 kinds of games?  
 ii) How many watch exactly one of the sports languages?  
 iii) Draw Venn Diagram showing results of the survey.

c) Find the solution to the recurrence relation

(08 M)

$$a_n = 6a_{n-1} + 11a_{n-2} - 6a_{n-3} \text{ given } a_0 = 20, a_1 = 5 \text{ and } a_2 = 15$$

Q.5 a) Show that if every element in a group is its own inverse, then the group must be abelian. (04M)

b) Explain Pigeonhole principle and Extended Pigeonhole Principle. Show that if 7 colors are used to paint 50 bicycles, at least 8 bicycles will be of same color. (08M)

c) i) Prove by mathematical induction  $x^n - y^n$  is divisible by  $x - y$ . (04 M)

ii) Consider the group  $G = \{1, 2, 3, 4, 5, 6\}$  under multiplication modulo 7 (04 M)

a) Find multiplication table of G

b) Find inverse of every element

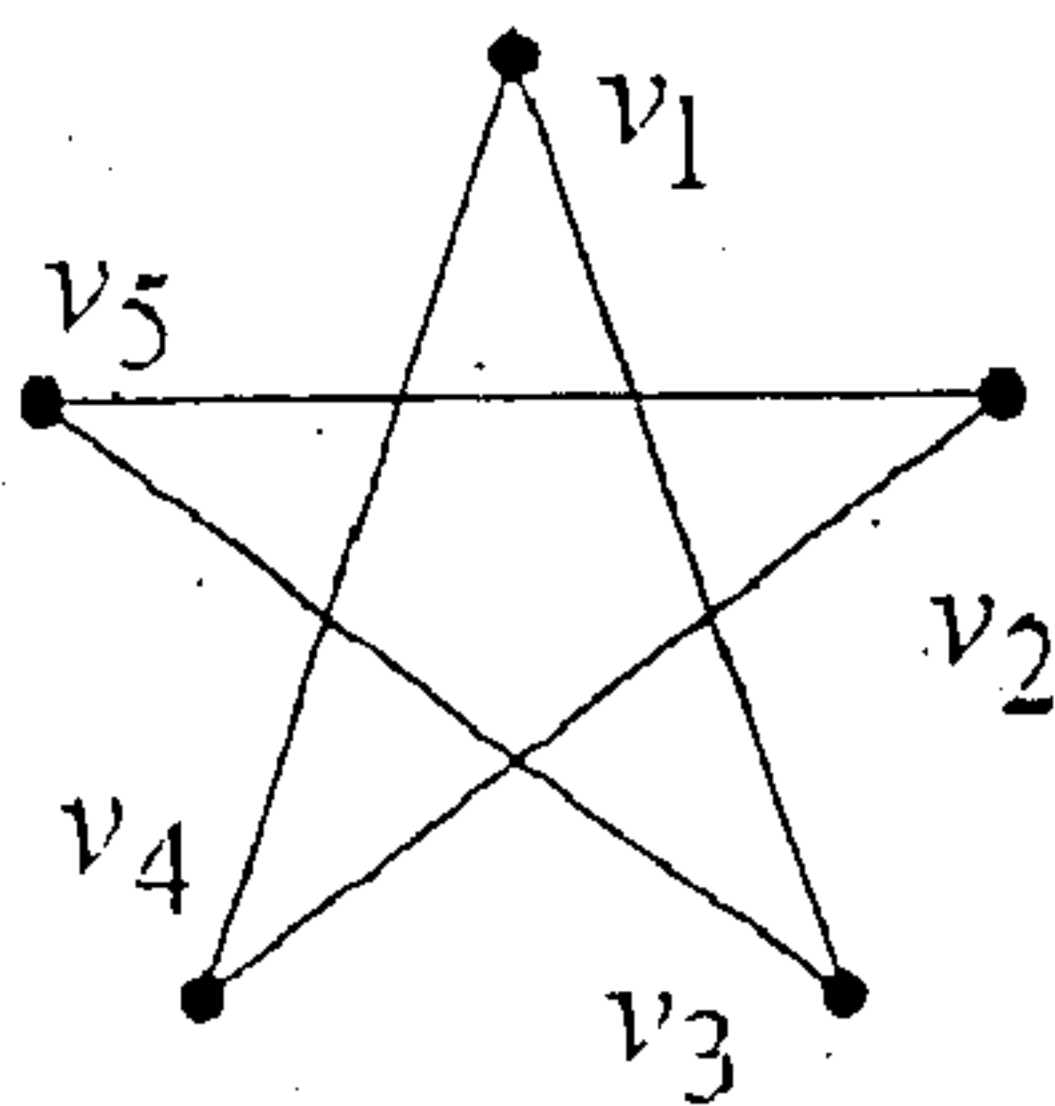
Q.6 a) Show that  $A - (B - C) = (A - B) \cup (A \cap B \cap C)$  (04 M)

b) Let  $H =$

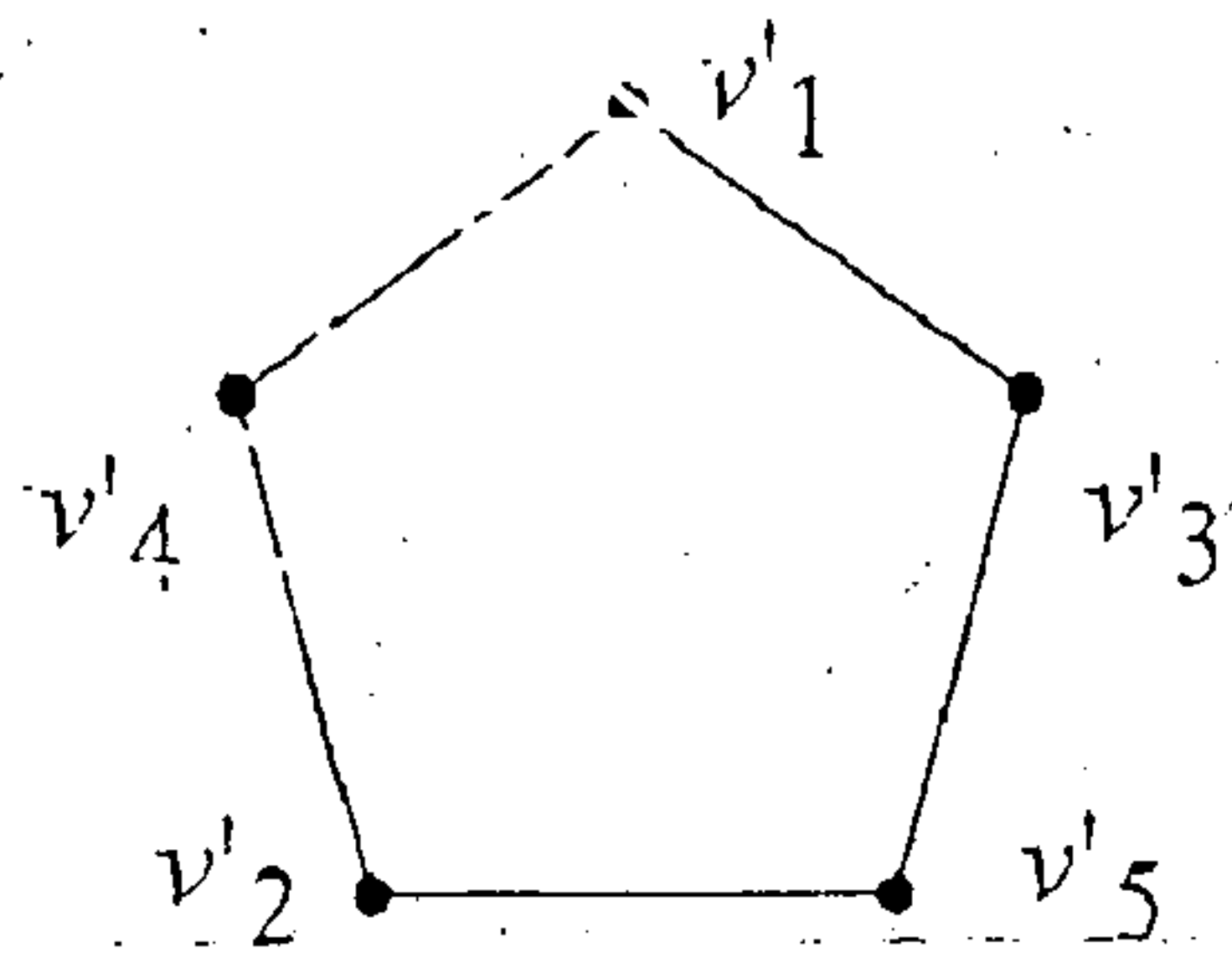
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Be a parity check matrix. Determine the group code  $e_H: B^3 \rightarrow B^6$  (08M)

c) Determine if following graphs  $G$  and  $G'$  are isomorphic or not. (08M)



$G$



$G'$