

(3 hours)



Total Marks: 80

N.B.

1. Question No 1 is compulsory
2. Solve any three question out of remaining five questions
3. Assumption made should be clearly stated
4. Figure to the right indicates full marks

1. (a) If function f is an isomorphism from semigroup $(S, *)$ to $(T, *')$, then prove that f^{-1} is an isomorphism from $(T, *')$ to $(S, *)$ 5
- (b) Find the generating function for the following sequence 5
 - i) 1, 2, 3, 4, 5,
 - ii) 0, 0, 0, 3, 3, 3, 3,
- (c) Find the truth table of $(p \Rightarrow q) \wedge ((q \Rightarrow r) \Rightarrow p)$ 5
- (d) For $x, y \in \mathbb{Z}$, xRy if and only if $2x + 5y$ is divisible by 7. Is R an equivalence relation? 5

2. (a) The college catering service must decide if the mix of food that is supplied for receptions is appropriate. Of 100 people questioned, 37 say they eat fruits, 33 say they eat vegetables, 9 say they eat cheese and fruits, 12 eat cheese and vegetables, 10 eat fruits and vegetables, 12 eat only cheese, and 3 report they eat all three offerings. How many people surveyed eat cheese? How many do not eat any of the offerings? 8
- (b) Prove by laws of logic (do not use truth table) that the following statement is a tautology 6

$$((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$$
- (c) Let $S = \{1, 2, 3, 4, 5\}$ and $A = S \times S$. Define the following relation R on $A(a, b) R (c, d)$ if and only if $ad = bc$. Show that R is an equivalence relation and compute A/R . 6

3. (a) Show that the set $G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ where the functions are defined by 8

$$f_1(x) = x \quad f_2(x) = 1 - x \quad f_3(x) = \frac{x}{x-1} \quad f_4(x) = \frac{1}{x}$$

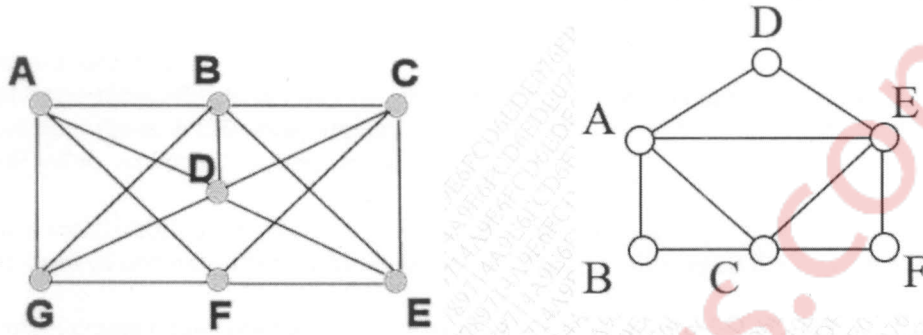
$$f_5(x) = \frac{1}{1-x} \quad \text{and} \quad f_6(x) = 1 - \frac{1}{x}$$

is a group under composition of two functions. Frame the multiplication table.
- (b) Solve $a_n - 7a_{n-2} + 6a_{n-3} = 0$, where $a_0 = 8$, $a_1 = 6$ and $a_2 = 22$ 6
- (c) Consider the lattices $L1 = \{1, 2, 4\}$, $L2 = \{1, 3, 9\}$ under divisibility. Draw the lattice $L1 \times L2$. 6

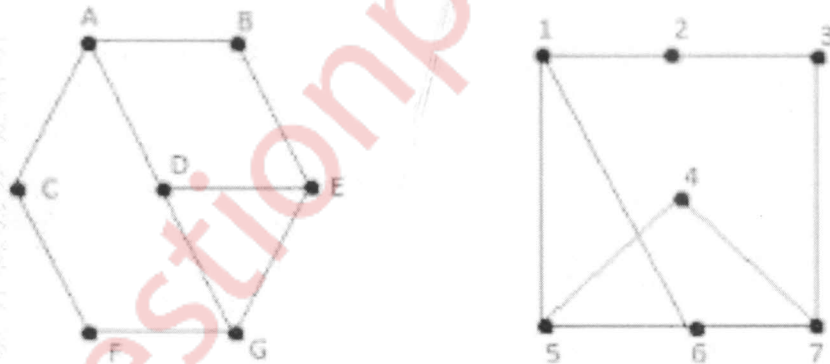
4. (a) Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be parity check matrix. 8

Determine the group code $e_H: B^2 \rightarrow B^5$

- (b) Prove the following using Mathematical induction $6^{n+2} + 7^{2n+1}$ is divisible by 43 6
- (c) If 11 numbers are chosen from a set $A = \{1, 2, 3, 4, \dots, 20\}$, then one of them is multiple of the other. 6
- 5 (a) Determine the Eulerian and Hamiltonian path/circuit, if any, in the following graphs. 8



- (b) Draw the Hasse diagram of the following poset (i) D_{72} (ii) D_{105} 6
- (c) Let $(G, *)$ is an Abelian group, then for all $a, b \in G$ show that $(a * b)^n = a^n * b^n$ (use mathematical induction) 6
6. (a) Determine whether following graphs are isomorphic 8



- (b) Solve the recurrence relation : $a_k - 3a_{k-1} = 2$ with initial conditions $a_0 = 1$ using generating function 6
- (c) A connected planar graph has g vertices having degree 2,2,2, 3,3,3, 4,4 & 5. How many edges are there? 6