

(3 Hours)

[Total Marks: 80]

- N.B.** (1) Question No 1 is compulsory  
 (2) Solve any three question out of remaining five questions  
 (3) Assumption made should be clearly stated  
 (4) Figure to the right indicates full marks



1. (a) Consider the set  $A = \{1, 2, 3, 4, 5, 6\}$  under the multiplication modulo 7. 08  
 (i) Find the multiplication table for the above  
 (ii) Find the inverse of 2, 3 and 5, 6  
 (iii) Prove that it is a cyclic group  
 (iv) Find the orders and the subgroups generated by  $\{3, 4\}$  and  $\{2, 3\}$
- (b) Determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. 06
- (c) Suppose that  $A$  is non empty set, and  $f$  is a function that has  $A$  as its domain. Let  $R$  be the relation on  $A$  consisting of all ordered pairs  $(x, y)$  where  $f(x) = f(y)$ . Show that  $R$  is an equivalence relation on  $A$ . 06
2. (a) Given  $S = \{1, 2, 3, 4\}$  and a Relation  $R$  on  $S$  given by 08  
 $R = \{(4, 3), (2, 2), (2, 1), (3, 1), (1, 2)\}$   
 (i) Show that  $R$  is not transitive  
 (ii) Find transitive closure of  $R$  by Warshall's algorithm
- (b) Show that  $n(n^2 - 1)$  is divisible by 24, where  $n$  is any odd positive integer.
- (c) Prove that a connected graph with  $n$  vertices must have at least  $n-1$  edges. 06  
 Can a single undirected graph of 8 vertices have 40 edges excluding self loop. 06
3. (a) Find the ordinary generating functions for the given sequences : 08  
 (i)  $\{0, 1, 2, 3, 4, \dots\}$  (ii)  $\{1, 2, 3, 4, \dots\}$   
 (iii)  $\{0, 3, 3^2, 3^3, \dots\}$  (iv)  $\{2, 2, 2, 2, \dots\}$
- (b) Functions  $f, g, h$  are defined on a set,  $X = \{1, 2, 3\}$  as [6] 06  
 $f = \{(1, 2), (2, 3), (3, 1)\}$ .  $g = \{(1, 2), (2, 1), (3, 3)\}$ .  $h = \{(1, 1), (2, 2), (3, 1)\}$ .  
 (i) Find  $f \circ g$ ,  $g \circ f$ , are they equal?  
 (ii) Find  $f \circ g \circ h$  and  $f \circ h \circ g$

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- (c) For each of the following sets of weights construct an optimal binary prefix code. For each weight in the set give the corresponding code word: 06
- (i) 1,2,4,6,9,10,12 (ii) 10,11,14,16,18,21 (iii) 5,7,8,15,35,40.
4. (a) Show that the (2,5) encoding function  $e: B^2 \rightarrow B^5$  defined by 08  
 $e(00)=00000$   $e(01)=01110$   
 $e(10)=10101$   $e(11)=11011$  is a group code. How many errors will it detect? 06
- (b) Prove the following  $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$  06
- (c) Let  $T$  be the set of all even integers. Show that  $(\mathbb{Z}, +)$  and  $(T, +)$  are isomorphic. 06
5. (a) Determine the matrix of the partial order of divisibility on the set 08  
 $A = \{1, 3, 5, 15, 30\}$ . Draw the Hasse diagram of the poset. Indicate whether it is a chain or not?
- (b) Define Hamiltonian path and circuit with example. what is the necessary and sufficient condition to exist Hamiltonian circuit? 06
- (c) Find the solution of  $a_{r+2} + 2a_{r+1} - 3a_r = 0$  that satisfies  $a_0 = 1, a_1 = 2$  06
6. (a) Determine whether the following posets are Boolean algebras. Justify your answers. 08
- (i)  $A = \{1, 2, 3, 6\}$  with divisibility
- (ii)  $D_{20}$ : divisors of 20 with divisibility
- (b) Define Universal and Existential quantifiers? Explain with examples. 06
- (c) Prove that the set  $G = \{0, 1, 2, 3, 4, 5\}$  is an Abelian group of order 6 with respect to addition modulo 6. 06

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COURSE : S.E (Sem - III) (CBSGS) (All Branches)  
(E)

QP Code : 541102

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Q3 a) Read as **find the generating functions.....**

ii) read as  $\{0, 3, 3^2, 3^3, \dots\}$

Q3 b) Ignore [6] marked as below

(b) Functions  $f, g, h$  are defined on a set,  $X = \{1, 2, 3\}$  as [6]

06

$f = \{(1, 2), (2, 3), (3, 1)\}$ ,  $g = \{(1, 2), (2, 1), (3, 3)\}$ ,  $h = \{(1, 1), (2, 2), (3, 1)\}$ .

(i) Find  $f \circ g$ ,  $g \circ f$ , are they equal?

(ii) Find  $f \circ g \circ h$  and  $f \circ h \circ g$

Q4 b) Read as "Prove the following  $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$ "

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