S. E. COMP SEM - III CBGS

Discrete Structure 16/12/13

28-11-13-DTP7-RM-3

Con. 9941-13. **GX-12185**

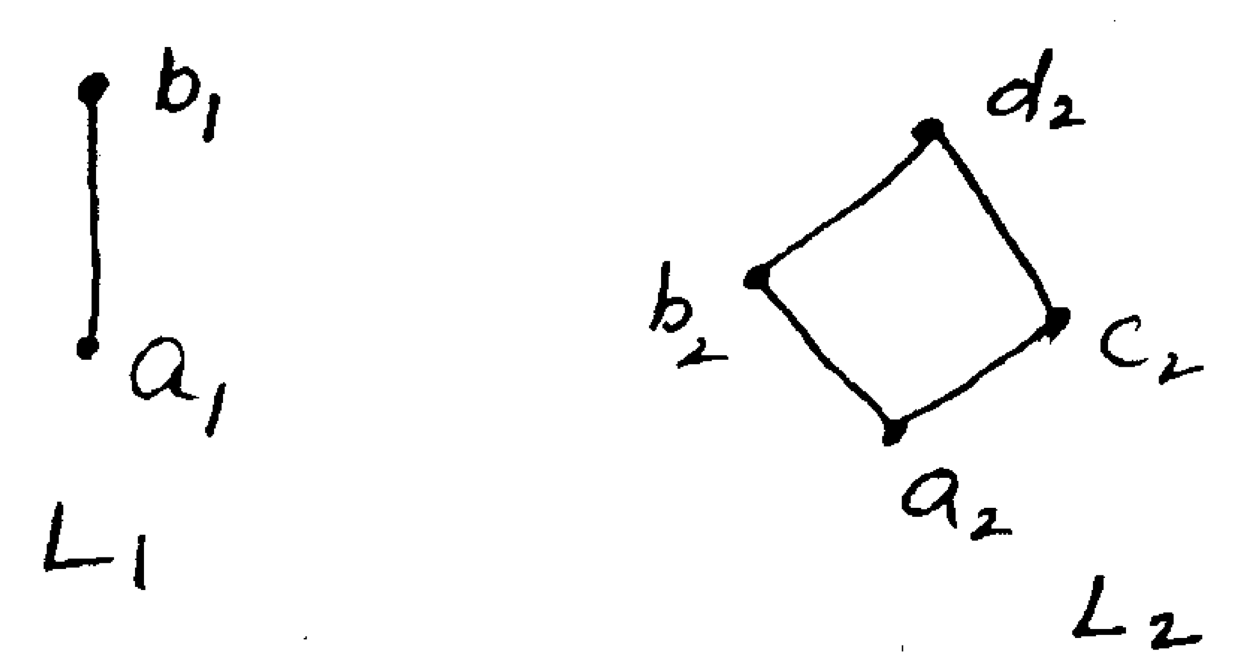
(3 Hours) **[Total Marks : 80**

N.B. : (1) Question no. **1** is **compulsory**.
(2) Attempt any **three** questions out of remaining **four** questions.
(3) Assumptions made should be clearly stated.
(4) **Figures** to the **right** indicate **full** marks.
(5) Assume suitable data wherever **required** and justify it.

1. (a) Prove that in a full binery tree with n vertices, the number of pendant vertices is $(n + 1) / 2$. **4**
(b) Let G be the set of rational numbers other than 1. Let define an operation $*$ on G by $a * b = a + b - ab$ for all $a, b \in G$. Prove that $(G, *)$ is a group. **6**
(c) Find the number of integers between 1 and 1000 which are **5**
(i) Divisible by 2, 3 or 5.
(ii) Divisible by 3 only but not by 2 nor by 5.
(d) Find all solutions of the recurrence relation **5**
$a_n = 5a_{n-1} + 6a_{n-2} + 7^n$

2. (a) Prove by mathematical induction $x^n - y^n$ is divisible by $x - y$. **4**
(b) Let m be the positive integers greater than 1. Show that the relation $R = \{(a, b) \mid a \equiv b \pmod{m}\}$, i.e. aRb if and only if m divides a-b, is an equivalance relation on the set of integers. **6**
(c) Let $s = \{1, 2, 3, 4\}$ and $A = S \times S$. Define the following relation :- **6**
R on A : $(a, b) R (a', b')$ if and only if $a + b = a' + b'$.
(i) Show that R is an equivalence relation.
(ii) Compute A/R.
(d) If $f : A \to B$ be both one-to-one and onto, then prove that $f^{-1} : B \to A$ is also both one-to-one and onto. **4**

3. (a) Consider an equilateral triangle whose sides are of length 3 units. If ten points are chosen lying on or inside the triangle, then show that at least two of them are no more than 1 unit apart. **5**
(b) Let L_1 and L_2 be lettices shown below :- **7**



Draw the Hasse diagram of $L_1 \times L_2$ with product partial order.
(c) Let $A = \{a, b, c\}$. Show that $(P(A), \subseteq)$ is a poset. Draw its Hasse diagram. P(A) is the power set of A. **4**
(d) How many vertices are necessary to construct a graph with exactly 6 edges in which each vertex is of degree 2. **4**

[**TURN OVER**

4. (a) Show that if every element in a group is its own inverse, then the group must be abelian. **4**

   (b) If (G, $*$) is an abelian group, then for all a, $b \in G$, prove that by mathematical induction $(a * b)^n = a^n * b^n$. **5**

   (c) If f is a homorphism from a commutative group (S, $*$) to another group (T, $*'$), then prove that (T, $*'$) is also commutative. **4**

   (d) Consider the (3, 5) group encoding function **7**

   $e : B^3 \rightarrow B^5$ defined by

   e(000) = 00000 e(100) = 10011

   e(001) = 00110 e(101) = 10101

   e(010) = 01001 e(110) = 11010

   e(011) = 01111 e(111) = 11100

   Decode the following words relative to a maximum likelyhood decoding function.

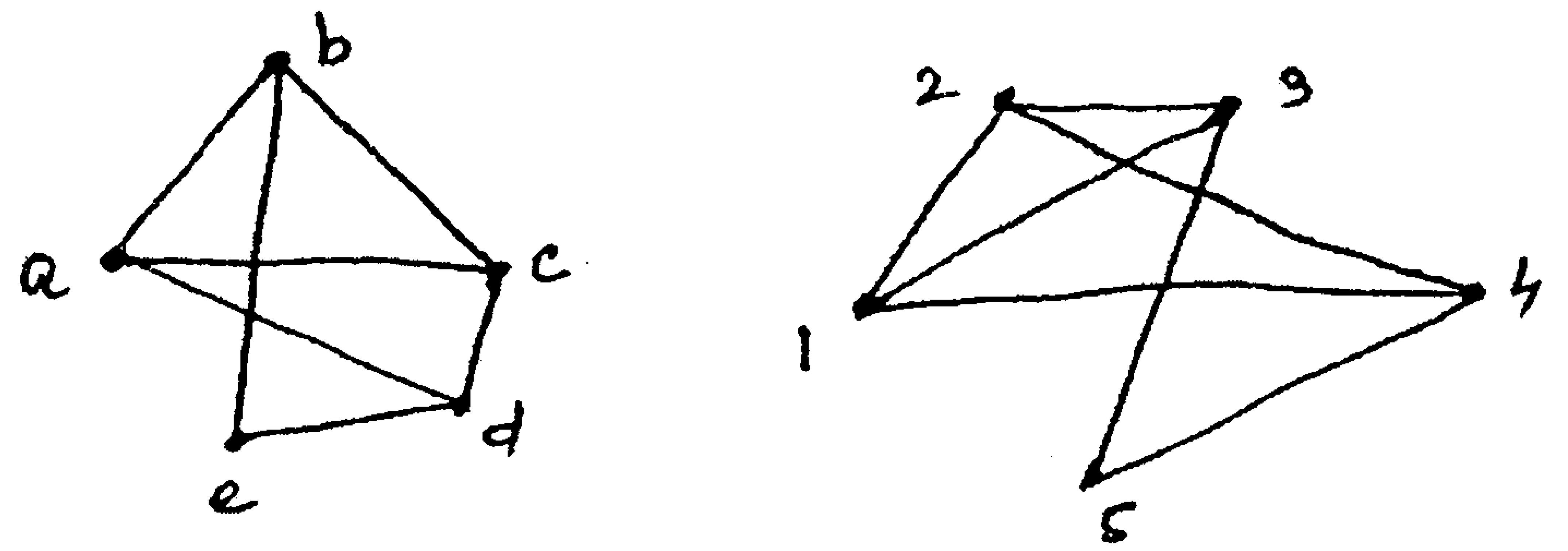
   (i) 11001 (ii) 01010 (iii) 00111

5. (a) Find the generating function for the following sequence **5**

   1, 2, 3, 4, 5, 6, ...

   (b) Solve the recurrence relation $a_r = 3a_{r-1} + 2$, $r \geq 1$ with $a_0 = 1$, using generating function. **6**

   (c) Show that the following graphs are isomorphic **5**



   (d) Use the laws of logic to show that **4**

   $[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$

   is a tautology