[3hours]   Total Marks 80

N.B. (1) Question No 1 is compulsory

(2) Attempt any 3 of the remaining

(3) Use of statistical table is allowed

1. a) A variable X follows a Poisson distribution with variance 3. Find   (5)

   P(x=2) and P(x$\geq$ 2)

   b) Evaluate $\iint_s 9\,xi + 6\,yj - 10\,zk\,.d\overline{s}$ where s is surface of the sphere with radius 2,

   using Gauss divergence theorem.   (5)

   c) Ten individuals are chosen at random from a population and their heights are
   found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 inches.
   Discuss the suggestion that the mean height of the universe is 65 inches.   (5)

   d) Using Cayley- Hamilton Theorem find $2A^5 - 3A^4 + A^2 - 4I$ where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ (5)

2. a) A continuous random variable X has the probability density function   (6)
      $f(x) = kx^2 e^{-x}$, $x \geq 0$. Find k,mean and variance

   b) Ten school boys were given a test in statistics and their scores were

   recorded. They were given a months special coaching and a second test

   was given to them in the same subject at the end of the coaching period.

   Test if the marks given below give evidence to the fact that the students are

   benefitted by the coaching.

   Marks in Test I   : 70, 68, 56, 75, 80, 90, 68, 75, 56, 58

   Marks in Test II :   68, 70, 52, 73, 75, 78, 80, 92, 54, 55   (6)

   c) Two lines of regression are given by x+6y = 6 ,and 3x+2y =10

   calculate (i) mean   values of x and y, (ii) the coefficient of correlation and   (8)

   (iii) estimate y when x = 12

3 .a) It is known that the probability of an item produced by a certain machine will be
      defective is 0.05. If the produced items are sent to the market in packets of 20,
      find the number of packets containing (i) at least2, (ii) exactly2 and (iii) at most
      2 defective items in a consignment of 1000 packets using Poisson distribution   (6)

b) Use Stoke's theorem to evaluate $\int_c \overline{F} . d\overline{r}$ where $\overline{F} = yzi + zxj + xyk$ and c is the

boundary of the circle $x^2 + y^2 + z^2 = 1$, z = 0 (6)

c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

(8)

4. a) Out of 800 people 25% were literate and 300 had travelled beyond the

limits of the district. 40 % of the literates were among those who had not

travelled. Prepare a 2x2 table and test at 5% level of significance whether

there is any relation between travelling and literacy (6)

b) Compute rank correlation coefficient from the following

X : 10, 12, 18, 18, 15, 40

Y : 12, 18, 25, 25, 50, 25 (6)

c) The marks of 1000 students of a university are found to be normally

distributed with mean 70 and standard deviation 5. Estimate the number of

students whose marks will be (i) between 60 and 75 (ii) more than 75

(iii) less than 68 (8)

5.a) A machine is set to produce metal plates of thickness 1.5 cms with standard
deviation of 0.2 cms. A sample of 100 plates produced by the machine gave
an average thickness of 1.52 cms. Is the machine fulfilling the purpose? (6)

b) Using the method of Lagrange's multipliers solve the following N.L.P.P

Optimise $z = 6x_1^2 + 5x_2^2$

Subject to $x_1 + 5x_2 = 7$,

$x_1, x_2 \geq 0$ (6)

c) If the vector field $\overline{F}$ is irrotational find the constants a,b,c where $\overline{F}$ is given by
$\overline{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$. Find the scalar potential of $\overline{F}$ .Then
find the workdone in moving a particle in this field from (1,2,-4) to (3,3,2)
along the straight line joining these points (8)

6.a) Using Green's theorem evaluate $\int_c (xy + y^2)dx + x^2 dy$ where c is the closed curve

of the region bounded by $y = x, y = x^2$ (6)

b) Show that $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is derogatory and find its minimal polynomial (6)

c) Using the Kuhn-Tucker conditions solve the following problem (8)

Maximise $z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

Subject to $x_1 + x_2 \le 2$, $2x_1 + 3x_2 \le 12$

$x_1, x_2, x_3 \ge 0$

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