Total Marks: 80   **Time: Three hours**

1. Q1 is compulsory

2. Solve any three out of the remaining from Q.2 to Q. 6.

3. Figures on the right hand side indicate marks.

4. Use of statistical tables is allowed.

Q.1. a) A continuous random variable has P.D.F. $f(x) = kx^2(1 - x^3), 0 \le x \le$
1, and $f(x) = 0$, otherwise.. Find k and mean.   5

b) If A=$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ then prove that $A^{-1} = A^2 - 5A + 9I$.   5

c) By using Green's theorem evaluate   the integral over the square formed by
the line x=$\pm 1, y = \pm 1, \oint(x^2 + xy)dx + (x^2+y^2)dy$ 5

d) Calculate Karl Pearson's coefficient of correlation from the data.   5

x	3	5	4	6	2
y	3	4	5	2	6

2. a) Random sample of 900 items is found to have a mean of 65.3cm. Can it be
regarded as a sample from a large population whose mean is 66.2 cm. and
standard deviation 5cm. at 5% level of significance?   6

b) Use the Lagrange's method of multipliers to solve the NLPP, optimize

$Z= 6x_1^2 + 5x_2^2, subjected\ to\ x_1 + 5x_2 = 7, x_1, x_2 \ge 0$ 6

c) A vector field is given by $\bar{F} =(x^2 + xy^2)i + (y^2 + x^2y)j$, prove that $\bar{F}$ is
irrotational, find the scalar potential   8

Q3. a) If x is a Poisson variable such that, $p(x=1) = p(x=2)$.find $E(x^2)$ 6

b) Evaluate by using Stokes theorem, $\oint 3ydx + 4zdy + 6ydz$ where c is the
curve of the intersection of sphere $x^2 + y^2 + z^2 = 8z\ and\ z = x + 4$.   6

c) A die was thrown 132 times and the following frequencies were observed.
Test the hypothesis that the die is unbiased.   8

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Number obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Q.4 a) Obtain Spearman's coefficient of rank correlation from the given data. 6

x	32	55	49	60	43	37	43	49	10	20
y	40	30	70	20	30	50	72	60	45	25

b) Use Gauss's divergence theorem to evaluate, $\iint x^2 dydz + y^2 dzdx + 2z(xy - x - y)dxdy$ and S is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. 6

c) Using the Kuhn Tucker method solve the NLPP, Maximize $Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$ subjected to $x_1 + x_2 \le 2, 2x_1 + 3x_2 \le 12, x_1, x_2 \ge 0$ 8

Q.5. a) Show that the matrix A = $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ is diagonalizable. Find the transforming matrix and the diagonal matrix. 6

b) Regression lines are given by 6y= 5x +90, 15x =8y+130, $\sigma_x^2 = 16$,

Find mean for x and y, correlation coefficient between x and y, and σ_y^2. 6

c) The standard deviations calculated from two random samples of sizes 9 and 13 are 1.99 and 1.9. can it be regarded as a sample drawn from the normal populations with the same standard deviations? (Given: $F_{0.025} = 3.51, with\ dof = 8\ and\ 12, F_{0.025} = 4.20, with\ dof = 12\ and\ 8$) 8

Q6.a) Find A^{50} if A = $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ 6

b) The monthly salary x in a big organization is normally distributed, with mean Rs. 3000 and standard deviation Rs 250. What should be the minimum salary of a worker in this organization so that the probability that he belongs to top 5% workers? 6

c) The heights of six randomly chosen sailors are in inches: 63, 65, 68, 69, 71 and 72. The heights of ten randomly selected soldiers are 61, 62, 65,66,69,69,70,71,72 and 73. Discuss in the light that this data suggests that the soldiers are on an average taller than sailors. 8