SE – IV | App. Maths – IV / Mech + Prod

CIVIL 21-11-14

QP Code : 12404

[3 Hours] **[Total Marks: 80]**

N.B. (1) Question no. **1** is **compulsory.**
(2) Attempt any **three** of the remaining.
(3) Use of statistical table is allowed.

1. (a) Using Green's theorem evaluate. 5

$\int_c (xy+y^2)dx + x^2dy$ where c is the closed curve of the region bounded by $y=x$ and $y=x^2$.

(b) Use Cayley-Hamilton theorem to find $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ in terms of A where $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ 5

(c) A continuous random variable has probability density function $f(x) = 6(x - x^2)$ $0 \le x \le 1$ Find mean and variance. 5

(d) A random sample of 900 items is found to have a mean of 65.3cms. Can it be regarded as a sample from a large population whose mean is 66.2cms. and standard deviation is 5 cms at 5% level of significance. 5

2. (a) Calculate the value of rank correlation coefficient from the folowing data regarding marks of 6 students in statistics and accountancy in a test 6

Marks in Statistics:	40	42	45	35	36	39
Marks in Accountancy:	46	43	44	39	40	43

(b) If 10% of bolts produced by a machine are detective. Find the probability that out of 5 bolts selected at random atmost one will be defective. 6

(c) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ 8

is diagonalisable. Find the transforming matrix and the diagonal matrix.

3. (a) In a laboratory experiment two samples gave the following results. 6

Sample	size	mean	sum of squares of deviations from the mean
1	10	15	90
2	13	14	108

Test the equality of sample variances at 5% level of significance.

3. (b) Find the relative maximum or minimum of the function. 6
$z = x_1^2 + x_2^2 + x_3^2 - 6x_1 - 10x_2 - 14x_3 + 103$

(c) Prove that $\bar{F} = (y^2\cos x + z^3)i + (2y\sin x - 4)j + (3xz^2 + 2)k$ is a conservative field. Find the scalar potential for $\bar{F}$ and the workdone in moving an object in this field from $(0, 1, -1)$ to $(\frac{\pi}{2}, -1, 2)$ 8

4. (a) The weights of 4000 students are found to be normally distributed with mean 50kgs. and standard deviation 5kgs. Find the probability that a student selected at random will have weight (i) less than 45 kgs. 6
(ii) between 45 and 60 kgs.

(b) Use Gauss's Devergence theorem to evaluate 6
$\iint_s \bar{N} \cdot \bar{F}\, ds$ where $\bar{F} = 4x\hat{i} + 3y\hat{j} - 2z\hat{k}$ and s is the surface bounded by $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$

(c) Based on the following data, can you say that there is no relation between smoking and literacy. 8

	smokers	nonsmokers
Literates	83	57
Illiterates	45	68

5. (a) A random variable X follows a Poisson distribution with variance 3 calculate $p(X=2)$ and $p(X \geq 4)$ 6

(b) Use Stoke'stheorem to evaluate $\int_c \bar{F} \cdot d\bar{r}$ where $\bar{F} = x^2 i + xyj$ and c is the boundary of the rectangle $x=0, y=0, x=a, y=b$ 6

(c) Find the equations of the two lines of regression and hences find correlation coefficient from the following data. 8

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

6. (a) Two independent samples of sizes 8 and 7 gave the following results. 6

Sample 1:	19	17	15	21	16	18	16	14
Sample 2:	15	14	15	19	15	18	16	

Is the difference between sample means significant.

6. (b) If $A=\begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ find A^{50} 6

(c) Use the Kuhn-Tucker Conditions to solve the following N.L.P.P 8

Maximise $z=2x_1^2-7x_2^2+12x_1x_2$

Subject to $2x_1+5x_2\leq 98$

$x_1,x_2\geq 0$
