Q. P. Code: 25571

(3hours) [Total marks: 80]

N.B. 1) Question No. 1 is compulsory.
2) Answer **any Three** from remaining
3) Figures to the right indicate full marks

1. a) Verify Laplace equation for $u = \left(r + \frac{a^2}{r}\right) \cos\theta$. 5

b) Find Laplace transform of $f(t) = e^{-3t} \sin 2t \, . \cos 3t$. 5

c) Obtain Fourier series for $f(x) = x$ in $(-\pi, \pi)$. 5

d) Evaluate $\int_C (z^2 + 3z^{-4})\, dz$ where C is the upper half of the unit circle from (1,0) to (-1,0). 5

2. a) Obtain the Taylor's and Laurent series which represent the function $f(z) = \frac{z}{(z+1)(z-2)}$ in the regions, i) $|z| < 1$ ii) $1 < |z| < 2$ 6

b) Obtain Complex form of Fourier series for $f(x) = \cos hx$ in $(-\pi, \pi)$ 6

c) Using Laplace transform, solve the differential equation:

$\frac{dx}{dt} + 2x = \sin\omega t$, with $x(0) = 1$. 8

3. a) Solve $\frac{\partial^2 u}{\partial x^2} - 100 \frac{\partial u}{\partial t} = 0$ with $u(0,t) = 0, u(1,t) = 0, u(x,0) = x(1-x)$ taking $h = 0.1$ for three time steps up to $t = 1.5$ by Bender –Schmidt method. 6

b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$. 6

c) Obtain half range Fourier sine series for $f(x) = \begin{cases} x, & 0 < x \le \pi/2 \\ \pi - x, & \pi/2 \le x < \pi \end{cases}$ 8

Hence, prove that –

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots.$$

[TURN OVER]

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4. a) Find the orthogonal trajectory of the family of curves $2x - x^3 + 3xy^2 = c$ 6

b) Find the Fourier series for $f(x) = x|x|$ in $(-1, 1)$. 6

c) Find the inverse Laplace transform of:-

i) $F(s) = \frac{1}{s(s^2+16)}$, using Convolution theorem, ii) $F(s) = cot^{-1}(s+1)$. 8

5. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,

$u(0,t) = 0,\ u(5,t) = 100,\ u(x,0) = 20$ taking $h = 1$ for one-time step. 6

b) Find the image of the circle $|z| = 4$ in the z-plane under the transformation $w = z + 4 + 3i$. Draw the sketch. 6

c) Find the analytic function $f(z) = u + iv$ if

$$u - v = \frac{cos\, x + sin\, x - e^{-y}}{2\, cos\, x - e^{y} - e^{-y}}$$

when $f\left(\frac{\pi}{2}\right) = 0$. 8

6. a) Using Residue theorem, evaluate, $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta}$ 6

b) Using Laplace transform, evaluate $\int_0^{\infty} e^{-t}(1 + 3t + t^2)H(t-2)dt$ 6

c) A tightly stretched string with fixed end points $x = 0$ and $x = l$, in the shape defined by $y = kx(l - x)$ where k is a constant is released from this position of rest. Find $y(x,t)$, the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. 8

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